1) Design a fully simplified 3-bit mod 6 down counter with your choice of T, JK, or D flip-flops. The circuit decrements at each clock pulse, going through the sequence
0, 5, 4, 3, 2, 1, 0, 5, 4, 3, … .
Show the circuit diagram.

2) Consider the following page replacement algorithms: FIFO (first in first out) and LRU (least recently used). Logical memory has 10 pages (pages 0 .. 9), while physical memory consists of 4 frames (frames 0 .. 3). The page reference string begins with 2, 6, 5, 7 to fill the four frames. Each part begins from this same initial point.

On your solution page, show the 2 frame traces for each part.

a) Continue the page reference string with at most 3 additional terms where LRU will result in strictly fewer page faults than FIFO.

ref. str.:  2   6   5   7    _   _   _
---------------
2   2   2   2
   6   6   6
      5   5
        7
---------------
2   2   2   2
   6   6   6
      5   5
        7
---------------
L RU

b) Continue the page reference string with at most 3 additional terms where FIFO will result in strictly fewer page faults than LRU.

ref. str.:  2   6   5   7    _   _   _
---------------
2   2   2   2
   6   6   6
      5   5
        7
---------------
2   2   2   2
   6   6   6
      5   5
        7
---------------
L RU

ref. str.:  2   6   5   7    _   _   _
---------------
2   2   2   2
   6   6   6
      5   5
        7
---------------
2   2   2   2
   6   6   6
      5   5
        7
---------------
F I FO
3) Consider the Readers/Writers problem with a single writer. Any number of readers can examine a file and the writer is only allowed access when there are no active readers. Consider the following incorrect solution. The common variables and their initializations are given by:

```c
semaphore wrt=1;
int readcount=0;
```

Code for the writer and the readers:

```c
writer()
{
    wait(wrt);
    //Do the writing
    signal(wrt);
}

reader()
{
    if(readcount==0)wait(wrt);
    readcount++;
    //Do the Reading
    readcount--;
    if(readcount==0)signal(wrt);
}
```

Give an execution sequence where a reader and the writer have access at the same time.

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1) Given a possibly empty binary tree containing character data, write a function that returns the number of left children in the tree. The prototype for your function should be

```c
int LeftCount(TreeNode *ptr)
```

Global variables may not be used. Declare all data structures.

2) Given a possibly empty singly linked list, write a function that reverses the last 4 nodes of the list (without altering the earlier nodes). If the given list has fewer than 5 nodes, the entire list should be reversed. The prototype for your function should be

```c
void Reverse4(Nodetype *ptr)
```

3) Solve the recurrence relation \( T(n) = 2T(n/2) + (n - 1) \) where \( T(1) = 0 \) and \( n = 2^k \) for a nonnegative integer \( k \). Your answer should be a precise function of \( n \) in closed form. An asymptotic answer is not acceptable. Justify your solution.
1. A certain programming language $P$ defines a comment as delimited by /# and #/. Let the alphabet $\Sigma = \{a, b, /, #\}$ and let $C$ be the set of all comments that begin with /#, end with #/, and contain no intervening #/. The shortest legal string in $L$ is therefore /##/.

   a. (10 points) Give a deterministic finite automaton (DFA) that recognizes legal comments $C$ in the language $P$.
   b. (10 points) Write a context-free grammar (CFG) that generates legal comments $C$ in the language $P$.

2. Consider the language $L = \{<M> | M$ is a Turing machine that accepts the string $w = 0011\}$.

   a. (5 points) Is $L$ decidable or undecidable?
   b. (15 points) Prove your answer above using reducibility. You may assume that the following languages are known to be undecidable:

   $\text{HALT}_{TM} = \{<M, w> : M$ is a Turing machine that halts on $w\}$
   $\text{ATM} = \{<M, w> : M$ is a Turing machine that accepts $w\}$

   You may not use Rice’s Theorem.

3. For each decision problem listed below, answer:
   i. Is the problem in the class $\text{NP}$?
   ii. Is the problem $\text{NP-complete}$?

   Scoring: each correct answer given is +2, each incorrect answers given is -1, no answer given is 0]

   DO NOT GUESS!

   a. Given a graph $G$, does $G$ contains a 3-clique?

   (a 3-clique is a subgraph of $G$ that is fully connected or complete on 3 vertices)

   b. Given two integers $n$ and $m$, are $n$ and $m$ relatively prime?

   (two integers are relatively prime if their greatest common divisor is 1)
c. Given a graph $G$ and a number $k$, is the largest clique in $G$ of size $k$?
   
   (a clique is a subgraph of $G$ that is a complete graph)

d. Given a Boolean expression $E$, are there exactly two truth assignments that satisfy $E$?
   
   (a Boolean expression is satisfiable if some assignment of variables makes it true)

e. Given a set of students $N = \{s_1, s_2, \ldots, s_{|N|}\}$, a set of final exams $M = \{e_1, e_2, \ldots, e_{|M|}\}$, a mapping $f: N \to P(M)$ showing the specific subset of exams each student is taking, and a number $t$ of possible time slots for the exams, is it possible to schedule the exams into the $t$ time slots such that no student has two of his or her exams assigned to the same time slot?