CS 692: Theory Exam
Fall 2020

Answer any **TWO** of the three problems listed below. If you attempt all three, only questions one and two will be graded. Please show all work.

Q1. Answer the following questions. In all cases Σ = {a, b}. **(10 points each).**

a) Let G be the grammar below:

$S \rightarrow aS \mid aB$
$B \rightarrow bB \mid b$

Let L(G) be the language generated by grammar G.
Either prove that L(G) is regular by providing a regular expression that describes L(G) or disprove it by applying the pumping lemma.

b) Let G' be the grammar below:

$S \rightarrow aSb \mid ab$

Let L(G') be the language generated by grammar G'.
Either prove that L(G') is regular by providing a regular expression that describes L(G') or disprove it by applying the pumping lemma.

Note: If you are using the pumping lemma to disprove the regularity of anyone of the above cases, you first need to clearly define the lemma. Then use it to disprove.

Q2. Provide a **context-free grammar** for each of the following languages. In all cases Σ = {0,1}. **(each 10 points)**

a. **L= {w| w = w^R and |w| is even},** where **w = w^R** means w is palindrome (reads the same forward and backward)

b. **L= {w| w starts and ends with the same symbol}**

Q3) Answer the following questions. Please clearly explain each in detail. (10 points each)

a) Consider the Vertex Cover (VC) problem defined as follows:
A vertex-cover of an **undirected** graph G = (V, E) is a subset of vertices **V' ⊆ V** such that if edge (u, v) is an edge in E, then either **u** ∈ **V'** or **v** ∈ **V'** or both.

VC = {G, k : G = (V, E) is an undirected graph containing a vertex-cover of size k}.

Prove that Vertex Cover (VC) is in NP.

Please note: You are being asked to prove that VC is in NP. You do NOT need to prove that VC is NP-Complete.

b) How do you prove, in general, that a Problem X is NP-complete? Please give the steps and explain.