

**CS 692: Theory Exam  
Spring 2021**

Answer any **TWO** of the three problems listed below. If you attempt all three, only questions one and two will be graded. Please show all work.

Q1. Give context-free grammars generating the following languages over  $\Sigma = \{0,1,2\}$  (10 points each).

a)  $\{0^n 1^m 2^k \mid n, m, k \geq 0, \text{ and } n = m \text{ or } m = k\}$

b)  $\{0^n 1^m 2^k \mid n, m, k \geq 0 \text{ and } n + m = k\}$

Q2. Consider  $\Sigma = \{0,1\}$ :

a. (5 points) State the Pumping Lemma for regular languages. Explain clearly and completely.

b. (15 points) Prove whether or not the following language is a regular language:  $L = \{0^n 1^m \mid m < n\}$ . If you choose to disprove, you need to apply the pumping lemma.

Q3. Answer the following questions. Please clearly explain each in detail and show all your work.

a) How do you prove, in general, that a Problem X is in NP? Please give the steps and explain (4 points).

b) Consider the problem Q defined below. Prove whether or not problem Q is in NP (10 points).

**Problem Q:**

INSTANCE: An undirected graph  $G(V,E)$  and a positive integer  $k$ .

QUESTION: Does graph  $G$  have a subset  $C$  of nodes such that  $|C| = k$  and there exists an edge in  $E$  between every pair of nodes in  $C$ ?

c) How do you prove, in general, that a Problem Y is NP-complete? Please give the steps and explain (6 points).