CS 692 Capstone Exam, Algorithms Fall 2021.

Choose any 2 of the 3 problems. If you attempt all three, only questions one and two will be graded. Please show all work.

Full name: _____ Net ID:_____

Question 1) (20 points) For each function below with input argument n, determine the asymptotic number of "basic operations" that will be executed. Justify your answer for each case. **Note:** For the recursive functions, you should first write the corresponding recurrence relation. Then solve the recurrence relation to come up with the asymptotic bound.

$\theta(1)$ $\theta(\log n)$ $\theta(n)$ $\theta(n\log n)$ $\theta(2^n)$ $\theta(n\log n^2)$ $\theta(n^2)$ $\theta(n^3)$ $\theta(n!)$ Other? Please specify.

a)

```
void func(int n) {
    if(n>1)
    {
        func(n-1);
        Perform n basic operations;
    }//endif
}
```

b)

```
void func(int n) {
  if (n > 3)
        {
                func(n/4);
                func(n/4);
                func(n/4);
                func(n/4);
                Perform n basic operations;
        }//endif
}
```

c)

```
void func(int n) {
  if (n > 1)
        {
                func(n/2);
                func(n/2);
                Perform 1 basic operation;
        }//endif
}
```

d)

```
void func(int n) {
        int i=n;
        while (i>0)
        {
                Perform 1 basic operation;
                i=i/4;

        }//endwhile
}
```

Question 2)

a) **(8 points)** Explain how heap data structures are different from binary search trees (BSTs). Provide at least two main differences and explain each.

b) **(12 points)** Apply the Heap Sort algorithm to sort the following list in ascending (**non-decreasing**) order. In addition to drawing the tree step by step, you should draw the array after each step. Show all your work.

int list[]={5, 6, 9, 8, 2, 1}

Question 3) **(20 points)** Consider two singly sorted linked lists, L1 and L2, each of which is sorted in ascending (non-decreasing) order. Assume L1 has n entries and L2 has m entries, where n, m>=0. Each entry has two components: a key component of type int and the usual next link component.

a) **(15 points)** Write a C++ Merge function to merge two given lists L1 and L2 in-place. That means your code should result in a singly merged list sorted in ascending order without creating a new list. Your Merge function should return a pointer to the head node of the merged list.

Here is an example of how merging would work. Assume the first linked list L1 has 4->35->95 and the other linked list L2 has 1->7->20->35, your code will produce 1->4->7->20->35->35->95 without using extra space.

b) **(5 points)** Analyze the time complexity of your code in part (a) in the worst-case. Justify your analysis.

CS 692 Capstone Exam Theory Fall 2021

Choose any 2 of the 3 problems.

1). Consider $\Sigma_1 = \{a, b, c\}$:

a. (5 pts) State the Pumping Lemma for regular languages.

b. (5 pts) Is the following language regular or not?

$$L_1 = \{ a^m b^n c^p : m \geq n \geq p \geq 0 \}$$

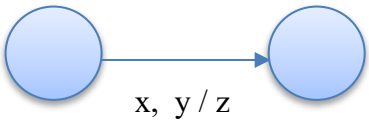
c. (10 pts) Prove your answer to question b. You may use Pumping Lemma if needed.

2). Consider the context-free language over $\Sigma_2 = \{x, y\}$:

$$L_2 = \{ x^n y^n : n \geq 0 \}$$

a. (10 pts) Give a context-free grammar for this language L_2.

b. (10 pts) Draw the state diagram of a pushdown automaton to recognize this language. You may use the following notation to label your machine's transitions:



(read input symbol x, stack top is y, push symbol z)

3). The SUBSET-SUM Problem takes as input a set S of integers and an integer T, the question is whether there exists a non-empty subset R that sums to T.

a. (5 pts) Define polynomial-time reducibility $A \leq_P B$.

b. (5 pts) In general, how do you prove that a given problem X is NP-Complete? Please list the steps.

c. (10 pts) Prove that 3-CNF-SAT $\leq_P$ SUBSET-SUM. (3-CNF-SAT problem: Given a formula in 3-CNF, is there an assignment of the variables such that the formula evaluates to true? For example, $(x \vee \neg y \vee \neg z) \wedge (\neg x \vee y \vee z) \wedge (\neg x \vee y \vee \neg z)$ is a 3-CNF formula.)

SYSTEMS EXAM

Fall 2021

90 minutes

Check which problems you are submitting:

- ❑ #1
- ❑ #2
- ❑ #3

How many pages total?_____

Please do not write on the back of any pages.

_____

(print name)

_____

(signature)

_____

(NetId)

1. **(20pts Total) Reader / Writer**

   Consider the Readers/Writers problem where any number of readers can examine a file, but only one writer at a time can update the file. A writer is only allowed access when there are no active readers. Consider the following code as a potential solution. The common variables are the two semaphores wrt and mutex. Both are initially set to 1, and the integer variable readcount is initially set to 0.

```
1:  semaphore wrt = 1, mutex = 1;
2:  readcount = 0;
3:
4:  writer()
5:  {
6:     wait(wrt);
7:            //writing is done
8:     signal(wrt);
9:  }
10:
11:  reader()
12: {
13:    wait(mutex);
14:           readcount++;
15:           if (readcount == 1) wait(wrt);
16:    signal(mutex);
17:        //Do the reading
18:    wait(mutex);
19:           readcount--;
20:           if (readcount == 0) signal(wrt);
21:    signal(mutex);
22: }
```

a) (4pts) Define the term "**race condition**" in the context of the **reader/writer** problem.

b) (2pts) Is this a **correct** solution to the reader writer/problem? (yes or no)

For **c)** and **d)** below **state** if the change will:

i) Have no significant effect
ii) Is needed for a correct solution
iii) Makes for an incorrect solution

For full credit you must explain your answer.

c) (7pts) What is the effect of swapping lines **15** and **16**?
d) (7pts) What is the effect of omitting lines **18** and **21**?

2. **(20pts Total) Memory Management – dynamic partitioning**

Given memory partitions of **500K, 300K, 600K** (in this order), how would each of the algorithms below place the following processes: **212K, 417K, 112K, 300K, 150K** (in this order). Please show your work. Memory can be partitioned

a) First-fit (4pts)

b) Best-fit (4pts)

c) Worst-fit (4pts)

d) (2pts) Which algorithm makes the most **efficient** use of memory in this case and why?

e) (2pts) What is **internal** fragmentation?

f) (2pts) What is **external** fragmentation?

g) (2pts) What is one **disadvantage** of both Best and Worst fit?

3. **(20 pts Total) Critical Section**

Consider the proposed solution of the critical section problem listed below. Common variables flag1, and flag 2 are initially false.

```
//Process 1
while (true)
{
       while (flag2);  //empty body
       flag1 = true;
       Critical section;
       flag1 = false;
       Noncritical section;
}
```

```
//Process 2
while (true)
{
    flag2 = true;
    while (flag1); //empty body
    Critical section;
    flag2 = false
    Noncritical section;
 }
```

a) (5pts) Define the terms **mutual exclusion**, **deadlock**, and **bounded waiting.**

b) (5pts) Does the code above guarantee **mutual exclusion**? If no, give an execution sequence where mutual exclusion is violated. If yes, give an explanation why all three requirements hold.

c) (5pts) Could **deadlock** occur? If no, explain why it cannot occur. If yes, give an execution sequence that leads to deadlock.

d) (5pts) Could **bounded waiting** occur? If no, explain why it cannot occur. If yes, give an execution sequence that allows bounded waiting.