CS 692 Capstone Exam Theory Fall 2021

Choose any 2 of the 3 problems.

1). Consider $\Sigma_1 = \{a, b, c\}$:

a. (5 pts) State the Pumping Lemma for regular languages.

b. (5 pts) Is the following language regular or not?

$$L_1 = \{ a^m b^n c^p : m \geq n \geq p \geq 0 \}$$

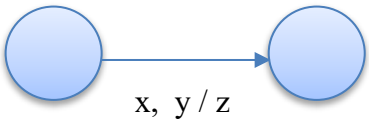
c. (10 pts) Prove your answer to question b. You may use Pumping Lemma if needed.

2). Consider the context-free language over $\Sigma_2 = \{x, y\}$:

$$L_2 = \{ x^n y^n : n \geq 0 \}$$

a. (10 pts) Give a context-free grammar for this language L_2.

b. (10 pts) Draw the state diagram of a pushdown automaton to recognize this language. You may use the following notation to label your machine's transitions:



(read input symbol x, stack top is y, push symbol z)

3). The SUBSET-SUM Problem takes as input a set S of integers and an integer T, the question is whether there exists a non-empty subset R that sums to T.

a. (5 pts) Define polynomial-time reducibility $A \leq_P B$.

b. (5 pts) In general, how do you prove that a given problem X is NP-Complete? Please list the steps.

c. (10 pts) Prove that 3-CNF-SAT $\leq_P$ SUBSET-SUM. (3-CNF-SAT problem: Given a formula in 3-CNF, is there an assignment of the variables such that the formula evaluates to true? For example, $(x \vee \neg y \vee \neg z) \wedge (\neg x \vee y \vee z) \wedge (\neg x \vee y \vee \neg z)$ is a 3-CNF formula.)