CS 692: Data Structures and Algorithms Capstone Exam, Spring 2022.

Choose any 2 of the 3 problems.

Full name: _________________ Net ID:_________________

Question 1) (10 points each)

Consider the following recurrence relations and solve them to come up with a precise function of n in closed form (that means you should resolve all sigmas, recursive calls of the function T, etc.). **An asymptotic answer is not acceptable here**. Justify your solution and show all your work.

a) $T(n) = 2T(n/2) + 7n$ where $T(1) = 1$ and $n = 2^k$ for a non-negative integer k.
b) $T(n) = 2T(n-1) + 1$ where $T(1)=1$

Question 2)

1. **(4 points)** Give a precise (formal) definition of $f(n) \in O(g(n))$ (“Big-Oh”).

2. **(4 points each)**

For each **function** f(n) **below**, give an **asymptotic upper bound** using “Big-Oh”. Choose from the following list (the list has no particular order):

$O(n^3)$, $O(n \log n)$, $O(n^4)$, $O(2^n)$, $O(1)$, $O(n)$, $O(\log n)$, $O(n \log n)$, $O(\log^3 n)$, $O(n^3 \log n)$, $O(n^n)$, $O(n!)$, $O(n^7 \log n)$, $O(n^2)$, $O(\log \log \log n)$

You should give the tightest bound possible. No need to justify your answer.

a) $f(n)= \log (7n^3) +16$
b) $f(n)= 2^n+ 10n^4+100$
c) $f(n)=n^2+ n \log n$
d) $f(n)=\left\{\begin{array}{ll} n^2 - 2n, & n \le 12 \\ 3n + 5, & n > 12 \ \ and\ n\ is\ odd \\ 12n, & n > 12 \ \ and\ \ n\ is\ even \end{array}\right\}$

Question 3) (20 points)

Implement (in C/C++) a **queue** of integers using **a singly linked list**. Declare the data structure and give code for the following operations:

a) empty_check, this operation checks whether the queue is empty or not
b) enqueue
c) dequeue (dequeue should both return a value and remove it from the queue)