1) a) Construct a circuit diagram for a 4x1 multiplexer.
   b) Let \( F(a, b, c, d) = a'b'c'd' + a'b'c'd + ab'c'd + ab'c'd. \) Use a 16x1 multiplexer (as a block diagram) and no additional logic gates to implement \( F. \)

2) Given the following 3 processes and main body that access the common variable i:

   ```c
   int i = 0;
   
   Processes:
   P1:                    P2:                        P3:
   i = i + 3;               i = i - 4;                 i = i + 5;
   
   Main body:
   Start asynchronously P1, P2, P3
   // After all 3 processes have finished
   Print i;
   
   a) What is the largest possible value of \( i \) that could be printed? Briefly explain how this can occur.
   b) List all other possible values of \( i \) that might be printed? (No explanations needed for part b).)

3) Consider a system with 3 resources (A, B, C) in quantity (10, 6, 9). The Banker’s Algorithm is used to allocate resources and it has the following SAFE state:

   Available:  A  B  C
               3  2  1

   Process     Allocation        Max        Need
               A  B  C        A  B  C        A  B  C
   P0              1   0   2        2   1   6         1   1   4
   P1              0   2   2        0   5   3         0   3   1
   P2              3   1   2        6   6   5         3   5   3
   P3              0   1   0        2   2   1         2   1   1

   a) Justify why the current state is safe.
   b) If P0 requests an additional unit of resource B, will it be allowed? Justify your answer.
Choose any 2 of the 3 problems.

1) Write the function int CountInternal(treeNode *p) that counts the number of non-leaf nodes in the (possibly empty) binary tree with root p. Declare all data structures.

2) Given a (possibly empty) singly linked list of distinct integers, write a function that removes the node containing the integer x. The function returns true if x is found, false otherwise. The prototype is

   bool remove_node(nodeptr & *head, int x);

3) For each function with input argument n, determine the precise number of “fundamental operations” that will be executed. Your answer should be a function of n in closed form. Note that “closed form” means that you must resolve all Σ’s and ⋯’s. An asymptotic answer (such as one that uses big-oh, big-theta, etc.) is not acceptable. Assume that n ≥ 1 for all parts.

   Note that fc is recursive.

a) void fa(int n) {
   for(int i = 0; i <= n; i = i+2)
     Perform 1 fundamental operation;
   //endfor i
}

b) void fb(int n) {
   for(int k = 2; k <= n; k++)
     for(int j = 1; j < n; j++)
       Perform 1 fundamental operation;
   //endfor j
   //endfor k
}

c) void fc(int n) {
   if (n > 1) {
     Perform n-1 fundamental operations;
     fc(n-1);
   } //endif
}
1. Give regular expressions describing each of the following languages over $\Sigma = \{0, 1\}$:
   a. $\{w : \text{the fourth symbol of } w \text{ is a } 0\}$
   b. $\{w : |w| \text{ is odd}\}$
   c. $\{w : w \text{ contains either substring } 000 \text{ or substring } 111\}$
   d. $\{w : \text{every } 0 \text{ in } w \text{ is immediately followed by a } 1\}$
   e. $\{w : |w| \neq 2\}$

2. Answer each of the following questions with **only YES or NO** to indicate whether or not the listed classes are closed under the indicated operations. *Do not guess if unsure, as wrong answers will lower your score!*
   
   **Scoring:** +2 points for correct answers; 0 points for no answers; -1 point for wrong answers
   a. regular languages under set difference
   b. regular languages under concatenation
   c. context-free languages under complement
   d. context-free languages under intersection
   e. decidable languages under union
   f. decidable languages under Kleene closure
   g. acceptable languages under complement
   h. acceptable languages under intersection
   i. co-acceptable languages under concatenation
   j. co-acceptable languages under set difference

3. A *clique* in an undirected graph is a subgraph wherein every two nodes are connected by an edge. Consider the language:

   $$\text{CLIQUE} = \{G, k : G = (V, E) \text{ is an undirected graph containing a clique of size } k\}$$

   Show that $3\text{SAT} \leq_p \text{CLIQUE}$