1) Design a fully simplified 3-bit mod 5 down counter with your choice of JK, D, or T flip-flops. The circuit decrements at each clock pulse, going through the sequence 0, 4, 3, 2, 1, 0, 4, 3, ... . Show the circuit diagram.

2) Consider two CPU scheduling algorithms for a single CPU: Preemptive Shortest-Job-First (also known as Shortest Remaining Time First) and Round-Robin. Assume that no time is lost during context switching. Given five processes with arrival times and expected CPU time as listed below, draw a Gantt chart to show when each process executes using
   a) Preemptive Shortest-Job-First (Shortest Remaining Time First).
   b) Round-Robin with a time quantum of 2.
   c) For the round-robin trace in part b, calculate the average turnaround time.
   Of course, assume that the expected time turns out to be the actual time.

<table>
<thead>
<tr>
<th>Process</th>
<th>Arrival Time</th>
<th>Expected CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>P2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>P3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>P4</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>P5</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>
3) Consider the Readers/Writers problem where any number of readers can examine a file, but only one writer at a time can update the file. A writer is only allowed access when there are no active readers. Consider the following code as a potential solution. The common variables are the two semaphores (both initially 1) and the integer variable readcount (initially 0).

```c
semaphore wrt=1,mutex=1;
readcount=0;

writer()
{
1:  wait(wrt);
   //writing is done
2:  signal(wrt);
}

reader()
{
3:  wait(mutex);
4:    readcount++;
5:   if(readcount==1)wait(wrt);
6:   signal(mutex);
   //Do the Reading
7:  wait(mutex);
8:    readcount--;
9:   if(readcount==0)signal(wrt);
10: signal(mutex);
}
```

In each of the following, your answer should be either
i) has no significant effect  or  ii) is needed for a correct solution
or  iii) makes for an incorrect solution

Explain your choice. Note that each question is independent. That is, it assumes the original code above.

a) What is the effect of swapping lines 5 and 6?
b) What is the effect of omitting lines 7 and 10?
1. Given a (possibly empty) binary tree of integers, write a recursive function that calculates and returns the number of leaf nodes in the tree. Declare all data structures.

2. Write a function which removes duplicate nodes in a (possibly empty) singly-linked list of integers that is already sorted in ascending order. That is, keep the first occurrence of each element value and remove any copies that follow. Do not create a new list, but instead re-link the original list. Runtime should be $O(n)$, where $n$ is the number of entries in the list.

3. Let $G$ be a connected undirected graph with $n$ vertices and $m$ edges. The edges are stored using the adjacency lists implementation.
   a) Write a routine to perform a breadth-first-search traversal on $G$. For any local data structures you use (such as stacks, queues, etc.), you may assume that the basic operations already exist (and so you don’t need to write the code for pop, enqueue, etc.).
   b) State in big-theta terms the runtime of your routine as a function of $n$ and $m$. 

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CS 6901  Capstone Exam  Data Structures  Spring 2013:  Choose any 2 problems.
1. Give a state diagram for a pushdown automaton that recognizes the following context-free language over $\Sigma = \{a, b, c\}$:

$$\{a^b b^c : x + y = z \text{ and } x, y, z \geq 0\}$$

2. Prove that the set difference operator is closed under decidable languages.

3. Answer each of the following questions with only YES or NO to indicate whether the conclusion is always true. Do not guess if unsure, as wrong answers will lower your score!

**Scoring:** +2 points for correct answers; 0 points for no answers; -1 point for wrong answers

a. If $A \leq_p B$ and $A$ is NP-Hard, then $B$ is decidable.

b. If $A \leq \overline{B}$ and $B$ is acceptable, then $A$ is co-acceptable.

c. If $A \leq_p B$ and $A$ is NP-Complete, then $B$ is decidable.

d. $A \leq B$ and $B$ is not decidable, then $A$ is not decidable.

e. If $A \leq B$ and $B$ is NP-Complete, then $A$ is NP-Hard.

f. If $A \leq_p B$ and $B$ is in NP, then $A$ is acceptable.

g. If $A \leq B$ and $B$ is decidable, then $A$ is acceptable.

h. If $A \leq_p B$ and $B$ is NP-Complete, then $A$ is in NP.

i. If $A \leq B$ and $B$ is in $P$, then $A$ is in $P$.

j. If $A \leq_p \overline{B}$ and $B$ is in NP, then $A$ is in NP.