Problem for 2001 January

Communicated by Dan Jurca
The following occurs as problem 2.5.17 in
Problems in Mathematical Analysis I
by W. J. Kaczor and M.T. Nowak.

Let

$$a_n = 3 - \sum_{k=1}^{n} \frac{1}{k(k+1)(k+1)!}, \quad n \in \mathbb{N}.$$ 

(a) Show that \( \lim_{n \to \infty} a_n = e. \)

(b) Show also that \( 0 < a_n - e < \frac{1}{(n+1)(n+1)!}. \)