The figure above shows one fourth of a circle of radius 1 centered at the origin of coordinates, a second circle tangent to this first circle and the positive coordinate axes, and finally a third circle tangent to the other two circles and the positive y-axis. What is the radius of this third circle?

Solution by Dan Jurca

Suppose the radius of the third circle is \( r \), the y-coordinate of the center is \( s \), and the radius of the second circle is \( t \). Then the center of the third circle is at \((r, s)\), the center of the second circle is at \((t, t)\), and we have the following equations.

\[
\sqrt{2} t + t = 1 \\
\sqrt{r^2 + s^2} + r = 1 \\
(t - r)^2 + (t - s)^2 = (r + t)^2
\]

Hence \( t = 1/(\sqrt{2}+1) = \sqrt{2} - 1 \), \( s = \sqrt{1 - 2r} \), and we have from the third equation \(-2tr + t^2 - 2ts + s^2 = 2tr\). Eliminating \( s \), we find the following.

\[
(9 - 4\sqrt{2})r^2 + (18 - 14\sqrt{2})r + (3 - 2\sqrt{2}) = 0
\]

Since one of the roots of this equation equals \( t = \sqrt{2} - 1 \) and the product of the two roots equals

\[
\frac{3 - 2\sqrt{2}}{9 - 4\sqrt{2}}
\]

we find that

\[
r = \frac{3 - 2\sqrt{2}}{(9 - 4\sqrt{2})(\sqrt{2} - 1)} = \frac{3 - 2\sqrt{2}}{17 - 13\sqrt{2}} \\
= \frac{3 - 2\sqrt{2}}{17 - 13\sqrt{2}} \cdot \frac{17 + 13\sqrt{2}}{17 + 13\sqrt{2}} \\
= \frac{1 - 5\sqrt{2}}{289 - 338} = \frac{1 - 5\sqrt{2}}{-49} = \frac{\sqrt{50} - 1}{50 - 1}.
\]

Also solved by Matthew Felix, Massoud Malek, Winston Teitler, and two others whose names and solution
I have lost (sorry — please resubmit)