Problem for 2012 August

Proposed by Dan Jurca

Prove that for each positive integer $k$ there exist positive integers $a$ and $b$ such that

$$a^2 + b^2 = (b + k)^2.$$ 

Solution by the proposer

For each positive integer $k$ if $a = 3k$ and $b = 4k$, then $a$ and $b$ are positive integers, and

$$a^2 + b^2 = (3k)^2 + (4k)^2$$
$$= 9k^2 + 16k^2$$
$$= 25k^2$$
$$= (5k)^2$$
$$= (4k + k)^2$$
$$= (b + k)^2.$$ 

Also solved by Massoud Malek