Let $R$ be a finite ring containing an element $r$ which is not a divisor of zero. Prove that $R$ must have a multiplicative identity.

Remark. One observes that the subring $2\mathbb{Z}$ of $\mathbb{Z}$, the integers, is an infinite ring with no nonzero zero divisor, and $2\mathbb{Z}$ does not contain a multiplicative identity.

Also solved by Arthur Fabian, Massoud Malek, and Winston Teitler