Problem for 1998 February

proposed by Dan Jurca

Find the sum of the first n terms of the following series.

$$1+2+4+7+11+16+22+29+37+46+56+\ldots$$

Solution by the proposer

Subtracting 1 from each term we get the series

$$0+1+3+6+10+15+21+28+36+45+55+\ldots,$$

so that the i-th term of the given series is the (i−1)-th ``triangular number", [((i−1)i)/2]. Hence the sum of the first n terms is

$$n+\sum_{i=1}^{n}\frac{(i-1)i}{2}=\frac{n^3+5n}{6}.$$