

Problem for 1998 May

proposed by Dan Jurca

Find a function $f:\mathbf{R}\rightarrow\mathbf{R}$ such that

$$0 \leq n \Rightarrow \lim_{x \rightarrow \infty} f^{(n)}(x) = \infty, \text{ and}$$

$$x \in \mathbf{R} \Rightarrow \lim_{n \rightarrow \infty} f^{(n)}(x) = 0$$

where $f^{(n)}$ is the derivative of order n of f .

Solution by the proposer

Consider $f:\mathbf{R}\rightarrow\mathbf{R}$ by $f(x)=e^{x/2}$. Then for $0 \leq n$ we have $f^{(n)}(x)=e^{x/2}/2^n$. Thus the function f obviously has the required properties.

Of course for each α , $0 < \alpha < 1$, the function $f(x)=e^{\alpha x}$ has the required properties.

No other solution was received.