

Problem for 1999 July

Proposed by Professor Emeritus Victor Manjarrez

Suppose that two triangles have the same area and also have the same perimeter. Does it follow that the triangles are congruent?

If so, give a proof; otherwise, give a counterexample.

Solution by Dan Jurca

We observe that

$$\begin{aligned}12^2+35^2 &=144+1225 \\ &=1369 \\ &=37^2, \quad \text{and} \\ 20^2+21^2 &=400+441 \\ &=841 \\ &=29^2;\end{aligned}$$

so that the isosceles triangle with base 24 and equal sides 37 has area $1/2 \times 24 \times 35 = 420$ and perimeter $24+37+37=24+74=98$; similarly the isosceles triangle with base 40 and equal sides 29 has area $1/2 \times 40 \times 21 = 420$ and perimeter $40+29+29=40+58=98$. The triangles are not congruent.

Also solved by Matt Hubbard and the proposer.