Problem for 2000 October

communicated by Dan Lindquist

Prove: If [a/(b–c)]+[b/(c–a)]+[c/(a–b)]=0, then
[a/((b–c)^2)]+[b/((c–a)^2)]+[c/((a–b)^2)]=0.

Solution by Professor Massoud Malek

From the first equation we have

$$\frac{a}{(b-c)^2} = -\frac{b}{(b-c)(c-a)} - \frac{c}{(b-c)(a-b)} = -\frac{b(a-b)+c(c-a)}{(a-b)(b-c)(c-a)} = -\frac{ab-b^2+c^2-ac}{(a-b)(b-c)(c-a)}$$

$$\frac{b}{(c-a)^2} = -\frac{a}{(c-a)(b-c)} - \frac{c}{(c-a)(a-b)} = -\frac{a(a-b)+c(b-c)}{(a-b)(b-c)(c-a)} = -\frac{a^2-ab+bc-c^2}{(a-b)(b-c)(c-a)}$$

$$\frac{c}{(a-b)^2} = -\frac{a}{(a-b)(b-c)} - \frac{b}{(a-b)(c-a)} = -\frac{a(c-a)+b(b-c)}{(a-b)(b-c)(c-a)} = -\frac{ac-a^2+b^2-bc}{(a-b)(b-c)(c-a)}$$

so that by adding we obtain the asserted result:

$$\frac{a}{(b-c)^2} + \frac{b}{(c-a)^2} + \frac{c}{(a-b)^2} = 0.$$

Also solved by Dan Jurca, Stu Smith, and Dangthu Ta.