Problem for 2004 December

Proposed by Bill Nico

Prove that the following construction of a regular pentagon is correct.

1. Draw a (unit) circle centered at O.
2. Draw a line (extended diameter) through O meeting the circle in points A and B.
3. At B construct a perpendicular to the line AOB and mark on it a point C such that length OB= length BC; *i.e.*, 1.
4. With center A and radius AC draw an arc meeting line AOB in point D (beyond B).
5. Bisect line segment OD, calling the midpoint E.
6. Construct the perpendicular bisector of segment OE, calling the midpoint F and the point where the perpendicular bisector meets the circle G.
7. The chord GB is one side of the pentagon inscribed in the circle; complete it in the obvious way.

Diagram

Solution by Dan Jurca

Since AD=AC=$\sqrt{5}$, we find OF=(AD−AO)/4=($\sqrt{5}$−1)/4, so that

$$\cos\angle BOG=\cos\angle FOG=(\sqrt{5}-1)/4=\cos(2\pi/5);$$

therefore BG is one side of a regular pentagon.

Also solved by Shavila Devi, Karen Nelson, and the proposer