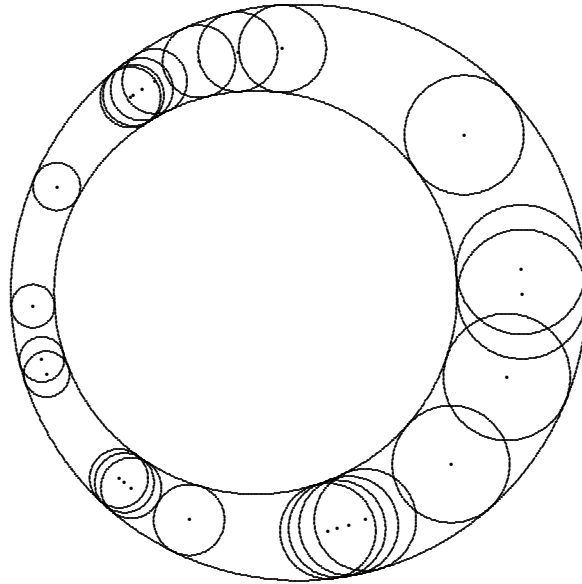


# Problem for 2007 December

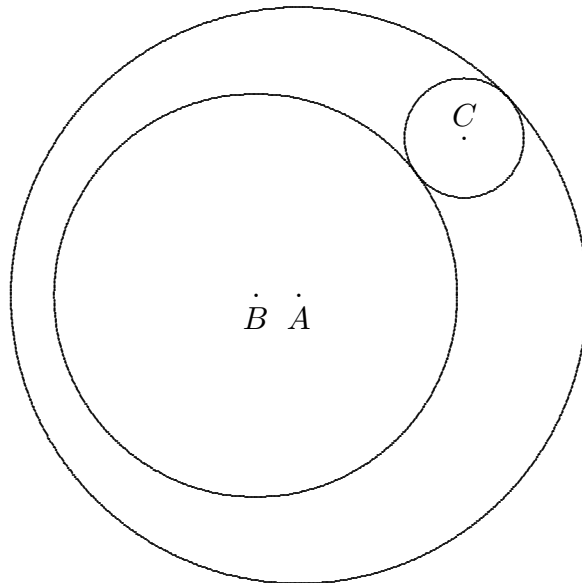
Proposed by Bill Nico

A smaller circle lies inside a larger circle, as in the sketch below. Find the locus of centers of circles lying between the two circles and tangent to both circles.



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Solution by Dan Jurca



With  $R$  the radius of the largest circle (centered at  $A$ ),  $r$  the radius of the smaller circle (centered at  $B$ ), and  $h$  the radius of a circle (centered at  $C$ ) tangent to these one sees that  $\overline{BC} = r+h$ , and  $\overline{AC}+h = R$ ; therefore  $\overline{AC} = R-h$ , so that  $\overline{AC}+\overline{BC} = (R-h)+(r+h) = R+r$ . Hence the sum of the distances from the centers of the circles tangent to the two given circles to the centers of the two given circles equals  $R+r$ , a constant. Thus the centers of the circles lying between and tangent to the two given circles lie on an ellipse with foci at the centers of the two given circles.

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Also solved by the proposer