

Problem for 2009 December

Proposed by Dan Jurca

Ramanujan famously remarked (to Hardy) that 1,729 is the least integer which equals the sum of two positive cubes in two different ways.

$$1,729 = 1^3 + 12^3 = 9^3 + 10^3$$

Find an integer which equals the sum of two positive cubes in three different ways.

Solution by Bojan Bašić (Serbia) and Murray Stokely

$$\begin{aligned} 87,539,319 &= 167^3 + 436^3 \\ &= 228^3 + 423^3 \\ &= 255^3 + 414^3 \end{aligned}$$

Additional solutions found by Murray Stokely:

$$\begin{aligned} 119,824,488 &= 11^3 + 493^3 \\ &= 90^3 + 492^3 \\ &= 346^3 + 428^3; \\ 143,604,279 &= 111^3 + 522^3 \\ &= 359^3 + 460^3 \\ &= 408^3 + 423^3; \\ 175,959,000 &= 70^3 + 560^3 \\ &= 198^3 + 552^3 \\ &= 315^3 + 525^3; \\ 327,763,000 &= 300^3 + 670^3 \\ &= 339^3 + 661^3 \\ &= 510^3 + 580^3; \\ 700,314,552 &= 334^3 + 872^3 \\ &= 456^3 + 846^3 \\ &= 510^3 + 828^3; \\ 804,360,375 &= 15^3 + 930^3 \\ &= 198^3 + 927^3 \\ &= 295^3 + 920^3; \\ 958,595,904 &= 22^3 + 986^3 \\ &= 180^3 + 984^3 \\ &= 692^3 + 856^3 \end{aligned}$$

The proposer found only $119,824,488 = 11^3 + 493^3 = 90^3 + 492^3 = 346^3 + 428^3$.

Jan Van Delden (The Netherlands) found: http://en.wikipedia.org/wiki/Taxicab_number