

Problem for 2010 November

Proposed by Dan Jurca

Do there exist integers r , a , and b such that the circle with equation

$$x^2 + y^2 = r^2$$

intersects the ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

in precisely four points, each with integer coordinates?

Solution by the proposer

Yes, for example the circle with equation

$$x^2 + y^2 = 17^2$$

intersects the ellipse with equation

$$\frac{x^2}{25^2} + \frac{y^2}{10^2} = 1$$

in the points with coordinates $(15, 8)$, $(-15, 8)$, $(-15, -8)$, and $(15, -8)$.

Also solved by Matthew Felix and Massoud Malek.