

Problem for 2012 March

Communicated by Dan Jurca

A cylindrical hole of length 10 inches is drilled through the center of a solid ball. Determine whether this is enough information to compute the volume of the remaining solid; and if it is enough, then find the volume of the remaining solid.

Solution by Dan Jurca

More generally, suppose the length of the hole through the ball is ℓ . Suppose the radius of the ball is R and the radius of the hole is r ; then we find $r^2 + (\ell/2)^2 = R^2$, and the volume V of the remaining solid can be computed as follows, where $x^2 + y^2 = R^2$.

$$\begin{aligned} V &= 2\pi \int_0^{\ell/2} y^2 dx - \pi r^2 \ell \\ &= 2\pi \int_0^{\ell/2} (R^2 - x^2) dx - \pi r^2 \ell \\ &= 2\pi [R^2(\ell/2) - (\ell/2)^3/3] - \pi r^2 \ell \\ &= 2\pi [R^2 \ell/2 - \ell^3/24] - \pi r^2 \ell \\ &= \pi [R^2 \ell - \ell^3/12 - r^2 \ell] \\ &= \pi [(R^2 - r^2)\ell - \ell^3/12] \\ &= \pi [(\ell/2)^2 \ell - \ell^3/12] \\ &= \pi [\ell^3/4 - \ell^3/12] \\ &= \pi \ell^3/6 \end{aligned}$$

Then if $\ell = 10$ inches, we find $V = 1,000\pi/6 \text{ in}^3 = 500\pi/3 \text{ in}^3$.

Also solved by

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