Problem for 2017 September

Communicated by Dan Jurca

If $S = \{(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)\}$ is a set of n points in the plane, then the *center* of S is the point

$$\left(\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}, \frac{y_1 + y_2 + y_3 + \cdots + y_n}{n} \right).$$

Suppose $p(x, y)$ is a polynomial with real coefficients in two variables of degree n, and

$$C = \{(x, y) \in \mathbf{R}^2 \mid p(x, y) = 0\}.$$

Suppose ℓ_1, ℓ_2, and ℓ_3 are three parallel lines in the plane, each of which intersects the curve C in exactly n points. Prove that the centers of these sets of intersection points lie on a single line; *i.e.*, the centers of $\ell_1 \cap C$, $\ell_2 \cap C$, and $\ell_3 \cap C$ lie on a line.