Full name: ___________________       Net ID:_________________

Question 1) (10 points each)
Consider the following recurrence relations and solve them to come up with a precise function of n in closed form (that means you should resolve all sigmas, recursive calls of the function T, etc.). An asymptotic answer is not acceptable here. Justify your solution and show all your work.

a) \( T(n) = 2T(n/2) + 7n \) where \( T(1) = 1 \) and \( n = 2^k \) for a non-negative integer \( k \).

b) \( T(n) = 2T(n-1) + 1 \) where \( T(1)=1 \)

Question 2)
1. (4 points) Give a precise (formal) definition of \( f(n) \in O(g(n)) \) (“Big-Oh”).

2. (4 points each)
For each function \( f(n) \) below, give an asymptotic upper bound using “Big-Oh”. Choose from the following list (the list has no particular order):

- \( O(n^3) \)
- \( O(n \log n) \)
- \( O(n^4) \)
- \( O(2^n) \)
- \( O(1) \)
- \( O(n) \)
- \( O(n \log n) \)
- \( O(\log^n n) \)
- \( O(n^n) \)
- \( O(n!) \)
- \( O(n^7 \log n) \)
- \( O(n^2) \)
- \( O(\log \log \log n) \)

You should give the tightest bound possible. No need to justify your answer.

a) \( f(n) = \log (7n^3) + 16 \)
b) \( f(n) = 2^n + 10n^4 + 100 \)
c) \( f(n) = n^2 + n \log n \)
d) \( f(n) = \begin{cases} 
  n^2 - 2n, & n \leq 12 \\
  3n + 5, & n > 12 \text{ and } n \text{ is odd} \\
  12n, & n > 12 \text{ and } n \text{ is even} 
\end{cases} \)

Question 3) (20 points)
Implement (in C/C++) a queue of integers using a singly linked list. Declare the data structure and give code for the following operations:

a) empty_check, this operation checks whether the queue is empty or not
b) enqueue
c) dequeue (dequeue should both return a value and remove it from the queue)