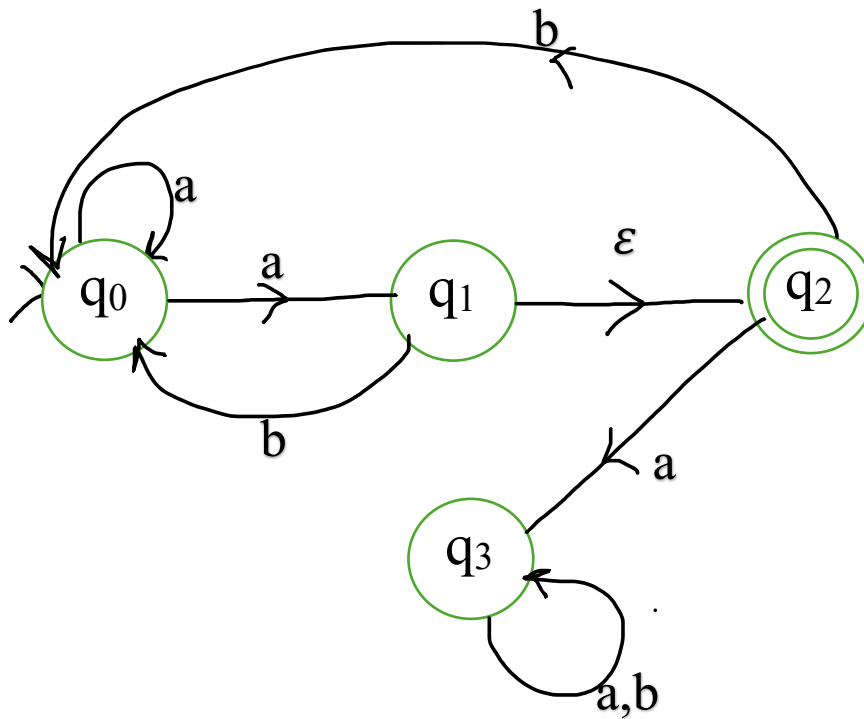**Choose any 2 of the 3 problems.**
If you answer all three questions, only questions 1 and 2 will be graded.

Full name: _____   Net ID:_____

Question 1) **(20 points)** Let $\Sigma = \{a, b\}$. Consider the following NFA. Note that state q_0 is connected to q_1 via an ε-move (epsilon-move).

Construct a **DFA** equivalent to the NFA provided above. Show all your work and explain how you constructed the DFA. Present your final answer as a **state diagram**.

- Be sure to present your final answer as a ****state diagram****, as no credit will be given for other forms of representation, including the transition table.

- Note that unclear drawings of any kind will not be awarded points.

Full name: _____   Net ID: _____

Question 2) Provide a **context-free grammar** for each of the following languages. In all cases $\Sigma = \{0,1\}$. **(10 points each)**

    a. $L = \{w | w = w^R$ and $|w|$ could be either odd or even$\}$, where $w = w^R$ means w is palindrome (reads the same forward and backward).

    b. $L = \{0^{3n} 1^n | n \geq 0\}$ (Here, **3n** is the exponent representing the number of **0**'s, and **n** is the exponent representing the number of **1**'s.)

Question 3) Answer the following questions. Please explain each answer clearly and in detail. **(10 points each)**

a) Consider the *Vertex Cover (VC)* problem defined as follows:
   A *vertex-cover* of an undirected graph **G = (V, E)** is a subset **V'⊆ V**, such that for every edge **(u, v)** ∈ **E**, at least one of **u** or **v** belongs to **V'**.

   Formally,
   VC = {(G, k) | G = (V, E) is an undirected graph that has a vertex-cover of size k}.

   Prove that Vertex Cover (VC) belongs to the complexity class **NP**. *(You are only asked to show VC* ∈ *NP. You do not need to prove NP-completeness.)*

b) In general, how do you prove that a decision problem X is NP-complete? State the standard steps of such a proof and explain.