CS 692: Capstone Exam
Theory Exam

Fall 2024, Nov 22, 2024 3:00-4:30pm

Instructions:

1. You can write your answers on **both sides of the paper**. If you run out of space as you answer the questions on the front pages, continue your answer on the back of the page. Please do not ask the instructor whether you can write on the back side of the paper. The answer is YES YOU CAN.
2. You may use the last paper sheet (front and back) as scratch paper. It is also marked as scratch paper. Please note that the instructor will NOT grade the scratch paper. The scratch sheet stays with the instructor. Please do not detach it.
3. Do NOT detach the exam sheets. If at any point the papers got detached, raise your hand and request for them to be stapled immediately.
4. You are NOT allowed to use Calculator.
5. You are NOT allowed to have your cell phone, e-watch, or other electronic devices nearby (on the desk or inside your pocket). They should be turned off and stay inside your bag or backpack which will be placed in the front or back of the room.
6. Exam duration: 3:00 to 4:30 pm.

Name	
NetID	

Check which TWO problems you are submitting:

☐	Problem 1
☐	Problem 2
☐	Problem 3

Problem 1. [Category: Design] Consider the language $L = \{0^i 1^j 2^k, i, j, k \geq 0, i + k = j\}$ over $\Sigma = \{0, 1, 2\}$.

(a) Give the Context-Free Grammar for this language. You only need to write out the grammar's rules with **S** as the starting symbol. You do NOT need to give the formal full definition of the grammar. You do NOT need to justify your answer. **[8 points]**

(b) Give the state diagram of the PDA for this language. Please use the notation in either Figure 1 or Figure 2 to label the transitions in the PDA. You do NOT need to justify your answer. **[12 points]**

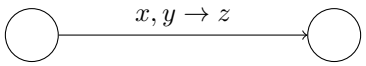

Figure 1: Notation 1: read input symbol x, stack top is y, push symbol z

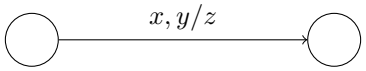

Figure 2: Notation 2: read input symbol x, stack top is y, push symbol z

Name: NetID:

Problem 2. [Category: Design] Consider $\Sigma = \{0, 1\}$

(a) Give the state diagram of the DFA that recognizes the language $L = \{w \mid w$ contains both 01 and 110 as substrings $\}$. You do NOT need to justify your answer. **[10 points]**

(b) Give the state diagram of the Deterministic One-tape Turing Machine that decides the language $L = \{w \mid w$ has as many 0s as 1s$\}$. You do NOT need to justify your answer.

For the Turing Machine, please use ONLY the following notation to label each of your machine's transitions where α, β can be the blank symbol $\sqcup$: **[10 points]**

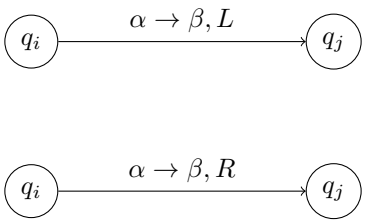

Figure 3: Notation format for Turing Machines

Problem 3. [Category: Proof]

A ***clique*** in an undirected graph is a subgraph, wherein every two nodes are connected by an edge. A ***k-clique*** is a clique that contains k nodes. The clique problem is to determine whether a graph contains a clique of a specified size. Let

$$CLIQUE = \{\langle G, k\rangle \mid G \text{ is an undirected graph with a } k\text{-clique}\}.$$

(a) Show that $CLIQUE$ is in NP. **[15 points]**

(b) What steps are needed to prove that CLIQUE is NP-complete? Just list the steps. A proof is not needed. **[5 points]**

Scratch paper Name: NetID: