bay area math collaborative

Pre-Calculus Course Guide

A COLLABORATION BY

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Introductions and Instructions

SO WHAT?

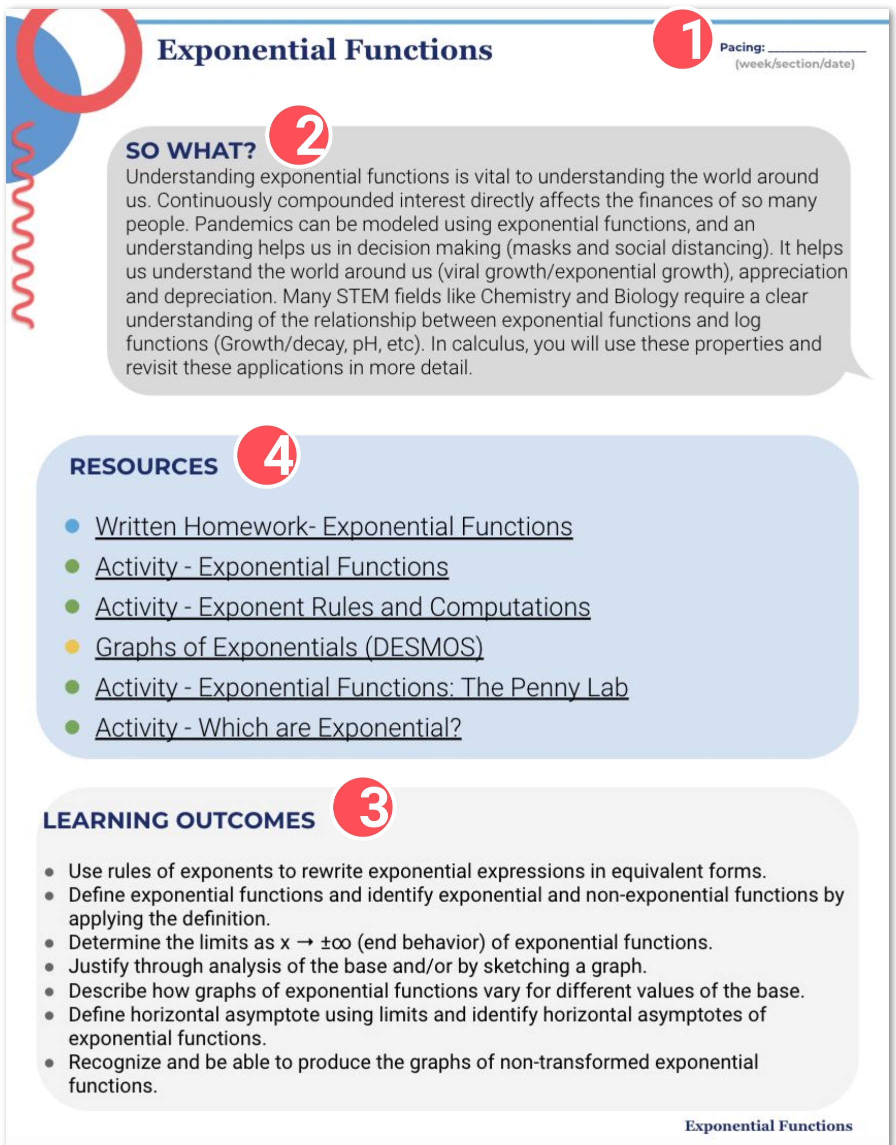
When learning new concepts, students will often ask, "so what?" A simple question that includes: Why is this lesson important? What does this have to do with real life? How is this useful to me? Why is the concept directly or indirectly useful in mathematics?
"So what?" helps teachers explain how mathematical concepts connect to the continuum of mathematical tools and can be applied to other STEM disciplines and everyday life

In this Course Guide, 3 key activity categories are utilized to organize our Resources:

WRITTEN HOMEWORK	DESMOS ACTIVITY	ACTIVITY / WORKSHEET

HOW TO USE:

1. Enter in the Pacing metric that you prefer to use to organize your course content. We recommend using weeks/dates or sections/units.
2. Review the course topic in the context of the "So What?" discussion to apply Big Ideas to real-world concepts..
3. Review the key Learning Outcomes for this topic.
4. Utilize the Resources in your classes to promote active engagement.



Exponential Functions

1 Pacing: ____________ (week/section/date)

SO WHAT? 2

Understanding exponential functions is vital to understanding the world around us. Continuously compounded interest directly affects the finances of so many people. Pandemics can be modeled using exponential functions, and an understanding helps us in decision making (masks and social distancing). It helps us understand the world around us (viral growth/exponential growth), appreciation and depreciation. Many STEM fields like Chemistry and Biology require a clear understanding of the relationship between exponential functions and log functions (Growth/decay, pH, etc). In calculus, you will use these properties and revisit these applications in more detail.

RESOURCES 4

- Written Homework- Exponential Functions
- Activity - Exponential Functions
- Activity - Exponent Rules and Computations
- Graphs of Exponentials (DESMOS)
- Activity - Exponential Functions: The Penny Lab
- Activity - Which are Exponential?

LEARNING OUTCOMES 3

- Use rules of exponents to rewrite exponential expressions in equivalent forms.
- Define exponential functions and identify exponential and non-exponential functions by applying the definition.
- Determine the limits as $x \to \pm\infty$ (end behavior) of exponential functions.
- Justify through analysis of the base and/or by sketching a graph.
- Describe how graphs of exponential functions vary for different values of the base.
- Define horizontal asymptote using limits and identify horizontal asymptotes of exponential functions.
- Recognize and be able to produce the graphs of non-transformed exponential functions.

Exponential Functions

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Functions and Real Numbers

Pacing: ______________
(week/section/date)

SO WHAT?

Functions are everywhere! From describing hospitalizations during the COVID pandemic with exponentials, to modeling waves with sines and cosines, functions are a fundamental tool for mathematicians. The definitions and precise language of functions allow us to interpret and predict processes that we observe around us. At this point we need to get some good examples of functions under our belts and agree on some vocabulary and notation to ensure that we don't have too many misunderstandings as we explain and describe functions.

RESOURCES

- Written Homework - Functions and Real Numbers
- Activity - Subsets of the real numbers
- Activity - Numerical Graphing
- Activity - Functions, Doman, and Range
- Activity - The Flag Hoist
- Activity - Root-Power - Radical Functions
- Activity - Functions in Tandem
- Activity - Pulling them Apart

LEARNING OUTCOMES

- Define function, domain, and range; use and interpret function notation correctly.
- Apply the definition of a function to written expressions and graphs; determine whether a relation is a function.
- Create examples and non-examples of functions.
- Determine domains of functions.
- State and apply the Vertical Line Test to determine if a graph represents a function.

Transformations of Graphs of Functions

Pacing: ______________
(week/section/date)

SO WHAT?

Functions are tools used to model real-life phenomena. As situations change, the functions we use in our models also change. Luckily, not every situation is brand new. Understanding transformations of functions gives us a general framework for adapting functions we know to new situations.

RESOURCES

- Written Homework - Transformations of Graphs of Functions
- Activity - Transformations Through Data
- Activity - Transformations of a Graph
- Transforming Functions (DESMOS) + Challenge (DESMOS)
- Activity - Transforming Graphs of Quadratic Functions
- Activity - Quadratic Puzzle
- Activity - Moving Other Functions Around
- Activity - More Functions to Move: Piece-by-Piece
- Activity - Composition or Transformation?
- Activity - Some Transformations of Trig Functions
- Activity - Back to Going in Circles
- Activity - More Problems

LEARNING OUTCOMES

- Recognize and be able to produce the graphs of these functions:
    - y = x, $y = x^2$ $y = x^3$, y = 1/x, y = √x, y = |x|, and y = c where c is a constant.Describe how the non-negative, real numbers A, C, and D in y = Af(x ± C) ± D affect the graph compared to the graph of y = f(x).
- Describe how the graphs of y = -f(x) and y = f(-x) compare to the graph of y = f(x).
- Given a function of the form y = ±Af(x ± C) ± D, use transformations to sketch its graph.

Polynomial Functions

Pacing: ______________
(week/section/date)

SO WHAT?

Polynomials can model real life phenomena like: projectile motion (such as water coming out of a hose), price/revenue/profit, simple interest, and area/volume. Polynomials are also the building blocks for other functions, like rational functions and conic sections. We can also use polynomials to approximate more complex functions! These properties make polynomials a great initial tool for solving problems.

RESOURCES

- Written Homework - Polynomial Functions
- Activity - Solving Polynomial Equations
- Activity - Quadratics
- Activity - Interpreting slope and intercepts
- Activity - Linear Functions Review
- Activity - Emails and Oil Spills
- Activity - Changing Forms
- Activity - Roots of Quadratic Functions
- Activity - Quadratic Puzzle
- Activity - Multiplying Lines

LEARNING OUTCOMES

- Define polynomial function and identify polynomial and non-polynomial functions by applying the definition.
- Determine the limits as $x \to \pm\infty$ (end behavior) of polynomial functions in both their factored and non-factored forms. Then justify this through analysis of the degree and the coefficient of the leading term.
- Use algebra to determine intercepts of polynomial functions.
- Use the long-run behavior (limits), intercepts, and continuity to roughly sketch polynomial functions in both factored and non-factored form.
- Recognize the relationship between factors and zeros/x-intercepts of a polynomial function.
- Use knowledge of lines to solve exercises about rates of change and knowledge of polynomials to solve exercises about optimization applications.

Exponential Functions

Pacing: ______________
(week/section/date)

SO WHAT?

Exponential functions can be used to model many real world phenomena. These models can then be used to help us answer questions in areas such as finance (compounding interest), infectious disease (pandemics), population growth and decay, and many other areas that impact our daily lives. STEM fields such as chemistry and biology use models that rely on exponential functions and their inverses, logarithmic functions. A solid understanding of the properties of exponentials will prepare you for further analysis in calculus.

RESOURCES

- Written Homework- Exponential Functions
- Activity - Exponential Functions
- Activity - Exponent Rules and Computations
- Graphs of Exponentials (DESMOS)
- Activity - Exponential Functions: The Penny Lab
- Activity - Which are Exponential?
- Activity: Solving exponential problem

LEARNING OUTCOMES

- Use rules of exponents to rewrite exponential expressions in equivalent forms.
- Define exponential functions and identify exponential and non-exponential functions by applying the definition.
- Determine the limits as $x \to \pm\infty$ (end behavior) of exponential functions.
- Justify through analysis of the base and/or by sketching a graph.
- Describe how graphs of exponential functions vary for different values of the base.
- Define horizontal asymptote using limits and identify horizontal asymptotes of exponential functions.
- Recognize and be able to produce the graphs of non-transformed exponential functions.

Inverse Functions

Pacing: ______________
(week/section/date)

SO WHAT?

Inverse functions help us to "undo" or "reverse" a process. For example, maybe we have a function that tells us the number of bacteria present at time t in a dish. The inverse of that function will help us answer the question, "How long does it take to double the amount of bacteria we started with?" Inverses swaps the input and output of the original function. Finding and working with a function and its inverse is an important skill in Calculus.

RESOURCES

- Written Homework- Inverse Functions
- Activity - Inverses
- Activity - Undoing Functions
- Activity - Graphs of Inverse Functions
- Activity - Inverse Trig Functions
- Activity - Inverse Compositions
- Activity - Putting It All Together

LEARNING OUTCOMES

- Use algebra to determine the inverse function f^{-1} when given f(x).
- Define inverse function and be able to find inverses given a bean diagram.
- Define one-to-one functions and describe how they relate to inverse functions.
- State and apply the Horizontal Line Test to determine if a graph represents a one-to-one function.
- Restrict the domain of a function that is not one-to-one in order to define an inverse function over that restricted domain.
- Explain the relationship between the graphs of f and f^{-1}.

Logarithmic Functions

Pacing: ______________
(week/section/date)

SO WHAT?

The importance of exponentials in STEM (and beyond) means that the inverses of exponentials are also very useful. The inverse of an exponential is a logarithm! Understanding how to use exponentials to model something like bacteria growth, means also understanding how to use logarithms. A good foundation in how to use and manipulate logarithms will be helpful as we try to solve problems that involve exponentials.

RESOURCES

- Written Homework- Logarithmic Functions
- Activity - Logarithm Computations
- Activity - Log functions as inverses
- Activity - Comparing Growth
- Log Rules and Graphs (DESMOS)
- Activity - Undoing Exponential Functions
- Activity - Graphs of Logarithmic Functions
- Activity - How do logs work?

LEARNING OUTCOMES

- Define logarithmic functions as inverses of exponential functions.
- Use the definition and basic properties of logarithms to convert between logarithmic and exponential expressions.
- Recognize and be able to graph non-transformed and transformed logarithmic functions.
- Describe how graphs of logarithmic functions vary for different values of b (the base).
- Determine the limits (end behavior) of logarithmic functions.
- Define one-sided limits, recognize one-sided limit notation, and use the notation appropriately.
- Identify vertical asymptotes of logarithmic functions.

Solving Exponential and Logarithmic Functions

Pacing: ______________
(week/section/date)

SO WHAT?

This topic combines ideas about exponential, logarithmic, and inverse functions in the context of solving equations. Exponential and logarithmic equations can be used to model growth and decay of bacteria, measure and compare strength of earthquakes, and other applications. For the first time, students have to choose the most appropriate solution method for the problem in front of them. We develop intuition about how to select strategies by trying things and seeing whether or not they work. Students learn to use the concept of inverse functions to manipulate equations into a form that's easier to work with, and then check whether solutions 'make sense' in the applied context and given domain restrictions. Students need to learn the formulas well enough that they can apply them in Calculus, and develop the skill of remembering and recognizing strategies that can be used as intermediate steps when solving more complicated problems.

RESOURCES

- Written Homework- Logarithmic Functions
- Activity - Logarithm Computations
- Activity - Log functions as inverses
- Activity - Comparing Growth
- Log Rules and Graphs (DESMOS)
- Activity - Undoing Exponential Functions
- Activity - Graphs of Logarithmic Functions

LEARNING OUTCOMES

- Solve exponential equations by rewriting each side to have the same base and then setting the exponents equal.
- Use the fact that exponential and logarithmic functions are inverses of each other to solve exponential and logarithmic equations (paying attention to domain and range).
- Use properties of logarithms (such as the product rule, quotient rule, and power rule) to rewrite logarithmic expressions in equivalent forms and to solve logarithmic equations.

Rational Functions

Pacing: ______________
(week/section/date)

SO WHAT?

Rational functions are great examples to think about if we want to understand functions with more complicated domains. Because rational functions are made up of polynomials, we already know a lot about them, so this is a great opportunity to practice our analyzing skills. We can also use rational functions to learn about different sorts of limits–a key concept in Calculus.

RESOURCES

- Written Homework- Rational Functions
- Activity - Graphing Rational Functions
- Rational Functions (Desmos)
- Activity - Dividing Lines

LEARNING OUTCOMES

- Define rational function, and identify rational functions by applying the definition.
- Determine domains of rational functions, and explain the relationship between the domain and discontinuities of the function.
- Determine the intercepts of a rational function, noting that all x-intercepts must be in the domain.
- Use algebra and one-sided limits to determine limits of rational functions at the 'edges' of the domain.
  - ($-\infty$, ∞, $x=a$ for a not in the domain)
- If $x = a$ is not in the domain of a function $f(x)$, determine whether f has a vertical asymptote or a removable discontinuity (hole) at $x = a$.
- Use the end behavior (limits), intercepts, and continuity to sketch graphs of rational functions.

Trigonometry: Fundamentals

Pacing: ______________
(week/section/date)

SO WHAT?

Trigonometric functions are widely used in architecture, engineering, and the sciences. They are perfect for modeling phenomena that happen in cycles (or periods), and are also used in computer graphics to perform operations like rotations. In this section, we focus on the relationships between angles on a circle, lengths on a right triangle, and the definitions of the trigonometric functions.

RESOURCES

- Written Homework - Trig Part 1
- Activity - Arc Length
- Unit Circle (Desmos)
- Activity - Going in Circles
- Activity - Angles Measured in Lengths?
- Activity - Labeling Points on the Unit Circle

LEARNING OUTCOMES

- Define radians in terms of arc length and rotations around a circle. Given an angle in radians, determine the quadrant of the coordinate plane in which it lies.
- Define the unit circle. Describe the connection between the special right triangles and the values that appear on the unit circle.
- Recognize the common right triangles (45--45--90) and (30--60--90), know the relationships between their side lengths, and use them to find exact values of trig functions of the angles.
- Describe the relationship between an angle θ given in radians and its associated point on the unit circle. Use this relationship to define all six trig functions in terms of the angle θ.
- Use the symmetry of the Unit Circle and knowledge of what quadrant angles reside in, to evaluate trig functions at angles whose radian measures are integer multiples of $\pi/6$ and $\pi/4$.
- Use right triangle trigonometry (SOH-CAH-TOA) and the Pythagorean theorem to compute the sine, cosine, tangent, cosecant, secant, and cotangent of an angle in a triangle as ratios of side lengths of the triangle.

Trigonometry: Graphs

Pacing: ______________
(week/section/date)

SO WHAT?

We use trig graphs to model periodic behavior, for example modeling heart beats, the behavior of planets, or sound/electricity/light waves. The graphs of trig functions offer us an alternate way of seeing the relationship between angles and lengths.

RESOURCES

- Written Homework - Trig Part 2
- Activity - Trigonometric Functions Marble Slide (Desmos)
- Activity - Graphs of Sines and Cosines
- Activity - Some Transformations of Trig Functions
- Activity - Back to Going in Circles
- Activity - More Problems

LEARNING OUTCOMES

- Use the unit circle to explain the behavior of the sine and cosine as functions of the real numbers. Make connections between the unit circle and domain/range and intercepts of these functions.
- Recognize and be able to produce the graphs of y = sin (x) and y = cos (x). Use the graphs to find the period of each function. Also, recognize transformed versions of these graphs using the transformations presented earlier this semester.
- Use the sine and cosine functions to explain the behavior of tangent as a function of the real numbers.
- Use zeros of the sine and cosine functions to explain the domain and intercepts of the tangent function.
- Use one-sided limits to determine the limits of the tangent function at the points of discontinuity in the domain.

Trigonometry: Inverses

Pacing: _______________
(week/section/date)

SO WHAT?

Just like with exponentials and logarithms, to properly understand and be able to use the trig functions in applications, we must understand the trig inverses. Inverse trigonometric functions will deepen our understanding of the requirement for functions to have an inverses and will give us tools for solving trigonometric equations.

RESOURCES

- Written Homework - Trig Part 3
- Activity - Inverse Trigonometric Functions
- Activity - Inverse Compositions
- Activity - Identity Crisis?
- Activity - Identity Disguises

LEARNING OUTCOMES

- Define the inverse sine and inverse cosine functions including the domain and range of each.
- Use the definitions of inverse trigonometric functions to determine exact values of inverse trig functions whose outputs are integer multiples of $\pi/6$ and $\pi/4$ and $\pi/3$.
- Use the definitions of inverse trig functions to determine exact values of inverse trig and trig functions composed together.
- Solve equations involving trig functions and inverse trig functions. Determine whether an equation has multiple solutions, and justify using knowledge of trig functions and inverse trig functions.

Trigonometry: Identities and Solving Equations

Pacing: ______________
(week/section/date)

SO WHAT?

Working with trig identities helps develop our mathematical fluency. This includes a skill called substitution, where we may substitute an expression for another in order to rewrite the expression in a more useful form. Also, solving trigonometric equations adds another tool for solving problems involving periodic behavior. Both are important for developing many calculus tools that have practical applications.

RESOURCES

- Written Homework- Trig Part 4
- Activity - Putting It All Together
- Activity- Identity Crisis
- Activity - Some more problems
- Activity- Identity Disguises

LEARNING OUTCOMES

- Use common identities to find the value of the trigonometric functions at angles outside of the usual multiples of π/6 and π/4.
- Use common identities to solve trigonometric equations.
- Use strategies from algebra to solve trigonometric equations.