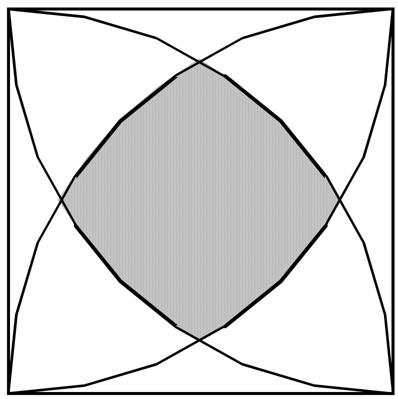
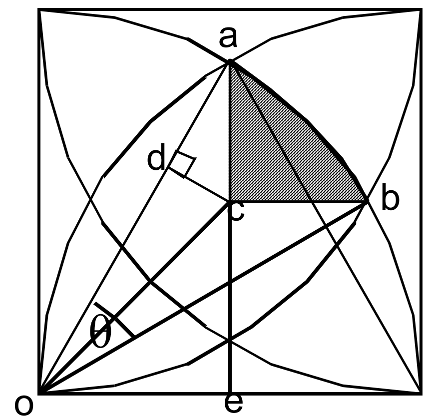
November 2005 Problem-of-the-Month Solution by Roger Doering

The length of each edge of a square equals 1; four circles of radius 1 are drawn, with centers at the corners of the square. Find the area of the region bounded by the four circles (shaded in the sketch).





We can inscribe an equilateral triangle into the figure which can be used to show that the angle θ is 30 degrees. Giving the area of the pie shaped wedge, (aob), as:

$$\text{wedge} := \frac{\pi}{12}$$

The length of the line segment ae is given by:

$$\text{ae} := \sqrt{1 - \left(\frac{1}{2}\right)^2} \quad \text{or} \quad \text{ae} := \frac{\sqrt{3}}{2}$$

This implies that the length of ac is : $\text{ac} := \frac{\sqrt{3}}{2} - \frac{1}{2}$

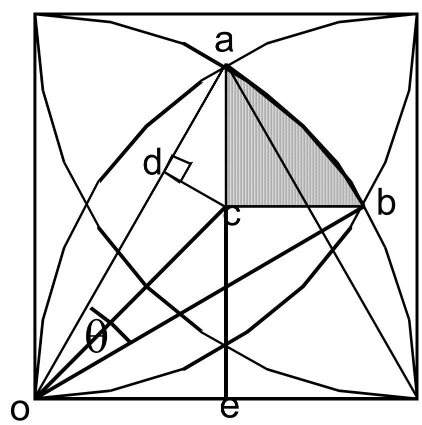
Since the angle dac is 30 degrees dc is half of the length of ac. $\text{dc} := \frac{\sqrt{3} - 1}{4}$

The area of the similar triangles aoc and boc is given by half of the line length of cd.
(use ao as the base of the triangle. ao = 1, and cd as the height of the triangle)

$$\text{Area}_{\text{aoc}} := \frac{\text{dc}}{2} \qquad \text{Area}_{\text{boc}} := \frac{\text{dc}}{2}$$

This gives the area of the slashed figure as : $\text{slash} := \text{wedge} - 2 \cdot \text{Area}_{\text{aoc}} \rightarrow \frac{1}{12} \cdot \pi - \frac{1}{4} \cdot 3^{\frac{1}{2}} + \frac{1}{4}$

The final area is four times that or: $\frac{\pi}{3} + 1 - \sqrt{3} = 0.31514674362772$


a
d
c
b
θ
o
e