

## Problem for 2011 December

Proposed by Dan Jurca

Suppose that the only operations a certain calculator can perform, apart from entering numbers, are multiplication and evaluation of square roots of nonnegative numbers. Show how this calculator can be used to evaluate cube roots of real numbers.

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Solution by the proposer

Since

$$\begin{aligned}\frac{1}{3} &= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \quad \text{it follows that if } 0 < N \text{ then} \\ N^{1/3} &= N^{1/4 + 1/16 + 1/64 + \dots} \\ &= N^{1/4} \times N^{1/16} \times N^{1/64} \times \dots \\ &= \sqrt{\sqrt{N}} \times \sqrt{\sqrt{\sqrt{\sqrt{N}}}} \times \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{N}}}}}} \times \dots, \quad \text{and since}\end{aligned}$$

$$\lim_{n \rightarrow \infty} N^{1/n} = 1$$

one can compute  $N^{1/3}$  using multiplication and repeated evaluation of square roots.

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Also solved by Kouros Ghaderi (who analyzed error) and Grant Morgan