Subject: Statistics
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Basic Probability: Key Definitions and Rules

1. **Key Probability Definitions and Notation**

- P**robability** is a number between 0 and 1 that is assigned to a possible outcome of a random circumstance.
- A **simple event** is a unique possible outcome of a random circumstance.
- A **compound event** is an event that includes two or more simple events.
- An **event** is any collection of one or more simple events in the sample space; events can be simple events or compound events.
- Events are often written using capital letters A, B, C, and so on, and their probabilities are written as *P(A), P(B), P(C),* and so on.
- One event is the **complement** of another event if the two events do not contain any of the same simple events and together they cover the entire sample space. For an event A, the notation A^c represents the complement of A.

  E.g. Event A: The roll of a die is 2;

  Event B: The roll of a die is NOT 2 (1 or 3-6).

  Event B is the complement of Event A, or $P(B) = P(A^c)$
- Two events are **mutually exclusive** if they do not contain any of the same simple events (outcomes). Equivalent terminology is that the two events are **disjointed.**

  E.g. Event A: The roll of a die is odd (1, 3, 5);

  Event B: The roll of a die is even (2, 4, 6).

  Event A and Event B are mutually exclusive.
- Two events are **independent** of each other if the probability of one occurring does not change the probability that the other occurs.

  E.g. Event A: Roll a die, and the outcome is 1;

  Event B: Roll another die, and the outcome is 1.

  Event A and Event B are independent.

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- Two events are **dependent** if knowing that one will occur (or has occurred) changes the probability that the other occurs.

  E.g. Event A: The roll of a die is odd (1, 3, 5);

  Event B: The roll of a die is 1.

  Event A and Event B are dependent.

- The **conditional probability of the event A, given that the event B has occurred or will occur,** is the long-run relative frequency with which event A occurs when circumstances are that B has occurred or will occur. It is written as ***P(A|B).***

  E.g. Event A: The roll of a die is odd (1, 3, 5);

  Event B: The roll of a die is not 1.

  P(A|B), or **the probability of Event A given B** means that given/assuming the roll of a die is not 1, the probability that it is odd (1, 3, 5).

2. **Probability Rules**

Rule 1 (for "not the event"): To find the probability of A^c, the complement of A, use

$$P(A^c) = 1 - P(A)$$

E.g. Event A: The roll of a die is 2; P(A) = 1/6

Event A^C: The roll of a die is NOT 2 (1 or 3-6). $P(A^c) = 1-1/6=5/6$

Rule 2 (addition rule for "either/or/and): To find the probability that either A or B or both happen:

- **Rule 2a (general):** ***P(A or B) = P(A) + P(B) – P(A and B)***
- **Rule 2b (for mutually exclusive events):** *If* A and B are ***mutually exclusive events:***

$$P(A \text{ or } B) = P(A) + P(B)$$

E.g. Event A: The roll of a die is odd (1, 3, 5); P(A) = 3/6 = 1/2

Event B: The roll of a die is even (2, 4, 6). P(B) = 3/6 = 1/2

Because Event A and Event B are mutually exclusive, P(A or B) = 1/2 +1/2= 1

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Rule 3 (multiplication rule for "and"): To find the probability that two events, A *and* B, both occur simultaneously or in a sequence:

- **Rule 3a (general):** ***P(A and B) = P(A)P(B|A) = P(B)P(A|B)***
- **Rule 3b (for independent events):** ***If*** A and B are ***independent events,***

  P(A and B) = P(A)P(B)

  E.g. E.g. Event A: Roll a die, and the outcome is 1; P(A) = 1/6

  Event B: Roll another die simultaneously, and the outcome is also 1. P(B) = 1/6

  Because Event A and Event B are independent, P(A and B) (meaning that roll two dice simultaneously, and both outcomes are 2) = 1/6 * 1/6= 1/36

- **Extension of Rule 3b to more than two independent events:** For several ***independent events:***

  $$P(A_1 \text{ and } A_2 \ldots \text{ and } A_{n-1} \text{ and } A_n) = P(A_1)P(A_2)\ldots P(A_{n-1})P(A_n)$$

  E.g. Event A_1: Roll *n* dice simultaneously, and the outcome of the first die is 2; $P(A_1)$ = 1/6

  Event A_2: Roll *n* dice simultaneously, and the outcome of the second die is 2. $P(A_2)$ = 1/6

  ....

  Event A_n: Roll *n* dice simultaneously, and the outcome of the *n*th die is 2. $P(A_n)$ = 1/6

  Because Event A_1, A_2 ... and A_n are independent of each other, P(A_1 and A_2 ... and A_{n-1} and A_n) = 1/6 * 1/6 ... 1/6*1/6= $1/6^n$

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Rule 4 (conditional probability): To find the probability that B occurs given that A has occurred or will occur:

$$P(B|A) = P(A \text{ and } B)/P(A)$$

The assignment of letters to events A and B is arbitrary, so it is also true that

$$P(A|B) = P(A \text{ and } B)/P(B)$$

E.g. Event A: The roll of a die is odd (1, 3, 5); P(A) = 1/2

Event B: The roll of a die is not 1. P(B)= 5/6

P(A and B), or the probability that the roll of a die is odd and not 1 = the probability that the roll of a die is 3 or 5 = 2/6 = 1/3

P(A|B) = P (A and B) / P(B) = (1/3) / (5/6) = 2/5

3. **Table for Reviewing the Rules:**

When Events Are:	**P(A or B) Is:**	**P(A and B) Is:**	**P (A|B) Is:**
Mutually Exclusive	*P(A) + P(B)*	*0*	*0*
Independent	*P(A) + P(B) –P(A)P(B)*	*P(A)P(B)*	*P(A)*
Any	*P(A) + P(B) – P(A)P(B|A)*	*P(A)P(B|A)*	*P(A and B)/P(B)*

References: The following works were referred to during the creation of this handout: *Mind on Statistics (4th Edition)*.