Basic Rules for Algebra & Logarithmic Functions

Quadratic Formula:	Example:
If $p(x) = ax^2 + bx + c$, $a \neq 0$ and $0 \leq b^2 - 4ac$, then the real zeroes of p are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.	If $p(x) = x^2 + 3x - 1$ =0, with a=1, b=3, and c=-1, then *p(x)* =0 if $x = \frac{-3 \pm \sqrt{13}}{2}$
Special Factors:	**Examples:**
$x^2 - a^2 = (x-a)(x+a)$	$x^2 - 9 = (x-3)(x+3)$
$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$	$x^3 - 8 = (x-2)(x^2 + 2x + 4)$
$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$	$x^3 + 27 = (x+3)(x^2 - 3x + 9)$
Binomial Theorem	**Examples:**
$(x+a)^2 = x^2 + 2ax + a^2$	$(x+3)^2 = x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9$
$(x-a)^2 = x^2 - 2ax + a^2$	$(x-5)^2 = x^2 - 2(x)(5) + 5^2 = x^2 - 10x + 25$
$(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$	$(x+2)^3 = x^3 + 3(x)^2(2) + 3(x)(2)^2 + 2^3 = x^3 + 6x^2 + 12x + 8$
$(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$	$(x-1)^3 = x^3 - 3(x)^2(1) + 3(x)(1)^2 - 1^3 = x^3 - 3x^2 + 3x - 1$
Factoring by Grouping	**Example:**
$acx^3 + adx^2 + bcx + bd = ax^2(cx+d) + b(cx+d) =$ $(ax^2 + b)(cx+d)$	$3x^3 - 2x^2 - 6x + 4 = x^2(3x-2) - 2(3x-2) =$ $(x^2 - 2)(3x-2)$

Arithmetic Operations:			
$ab + ac = a(b+c)$	$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$	$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
$a\left(\frac{b}{c}\right) = \frac{a}{1} \times \frac{b}{c} = \frac{ab}{c}$	$\frac{a-b}{c-d} = \left(\frac{-1}{-1}\right)\frac{a-b}{c-d} = \frac{b-a}{d-c}$	$\frac{ab+ac}{a} = \frac{a(b+c)}{a} = b + c$, $a \neq 0$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{b} \div \frac{c}{1} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$
$\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{1} \div \frac{b}{c} = \frac{a}{1} \times \frac{c}{b} = \frac{ac}{b}$			

Exponents and Radicals			
$a^0 = 1,\ a \neq 0$	$\frac{a^x}{a^y} = a^{x-y}$	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$\sqrt[n]{a^m} = a^{m/n} = \left(\sqrt[n]{a}\right)^m$
$a^{-x} = \frac{1}{a^x}$	$\left(a^x\right)^y = a^{xy}$	$\sqrt{a} = a^{1/2}$	$\sqrt[n]{ab} = \sqrt[n]{a} * \sqrt[n]{b}$
$a^x a^y = a^{x+y}$	$(ab)^x = a^x b^x$	$\sqrt[n]{a} = a^{1/n}$	$\sqrt[n]{\left(\frac{a}{b}\right)} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Algebraic Errors to Avoid:	
$\frac{a}{x+b} \neq \frac{a}{x} + \frac{a}{b}$	To see this error, let a, b, and, x all equal to 1. Then, $\frac{1}{2} \neq 2$.
$\sqrt{x^2 + a^2} \neq x + a$	To see this error, let $x = 3$ and $a = 4$. Then, $5 \neq 7$.
$a - b(x-1) \neq a - bx - b$	Remember to distribute the negative signs. The equation should be $a - b(x-1) = a - bx + b$
$\frac{\left(\frac{x}{a}\right)}{b} \neq \frac{bx}{a}$	To divide fractions, invert and multiply. The equation should be $\frac{\frac{x}{a}}{b} = \frac{\frac{x}{a}}{\frac{b}{1}} = \left(\frac{x}{a}\right)\left(\frac{1}{b}\right) = \frac{x}{ab}$
$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 - a^2}$	We can't factor a negative sign outside of the square root.
$\frac{a+bx}{a} \neq 1 + bx$	This is one of many examples of incorrect cancellation. By applying the properties above, the equation should be $\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$.
$\left(x^2\right)^3 \neq x^5$	In this case we multiply the exponents, then the correct equation is $(x^2)^3 = x^2x^2x^2 = x^{2+2+2} = x^6$

LOGARITHMIC RULES

Logarithmic function	$y = \log_a x$, defined by $x = a^y$
Common logarithm	$\log x = \log_{10} x$
Natural logarithm	$\ln x = \log_e x$
Log Properties	
	$\log_a a^x = x$; $a^{\log_a x} = x$
	$\log_a(MN) = \log_a M + \log_a N$
	$\log_a(M / N) = \log_a M - \log_a N$
	$\log_a(M)^N = N(\log_a M)$
Change of base formulas	$\log_b x = \frac{\log x}{\log b}$
	$\log_b x = \frac{\ln x}{\ln b}$

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Solving Quadratic Equations

Standard form of Quadratic Equation: $ax^2 + bx + c = 0$

1. *Completing the Square Method:* Example: $2x^2 - 10x + 9 = 0$

   Step 1: Make sure the coefficient of x^2 is 1, if it is not already 1 then divide both sides of the equation by a. For our example, we will divide by 2 on both sides, and we get: $x^2 - 5x + \frac{9}{2} = 0$.

   Step 2: Take the coefficient of x in the new equation in scratch, divide it by 2 and then square that term. So, it will look something like: $\left(\frac{b}{2a}\right)^2$, we get: $\frac{25}{4}$.

   Step 3: Now add $\left(\frac{b}{2a}\right)^2$ to both sides of the equation, and move the constant term on left side to the right side, so we get: $x^2 - 5x + \frac{25}{4} = \frac{25}{4} - \frac{9}{2}$.

   Step 4: Now you will notice that the left side of the equation becomes a perfect square, in our case we get: $\left(x - \frac{5}{2}\right)^2 = \frac{7}{4}$.

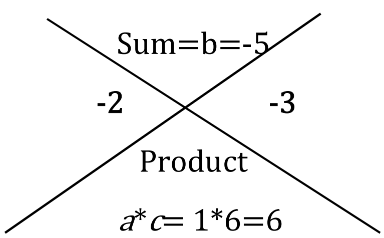
   Step 5: Now we will just solve this equation for x, and here we get: $x = \frac{5 \pm \sqrt{7}}{2}$.

2. *Factoring by grouping or Diamond Method*

Find 2 numbers that have the Sum of *b* and the Product of *a* times *c*. After you get the two numbers, you will get the factors (x + *constant 1*)*(x + *constant 2*)

Example: $x^2 - 5x + 6 = 0$

So we need to find 2 constants that have the sum of -5 and product of 6



$x^2 - 5x + 6 = 0$

$(x + (-2))*(x + (-3)) = 0$

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ or $x = 3$

References - The following works were referred to during the creation of this handout: Valle Verde Tutorial Support Service Handout and Basic Rules for Algebra Handout by Tomee Lu(SCAA Tutor).

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