

Limits to Infinity

Summary:

- $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$
- $\lim_{x \rightarrow \pm\infty} \frac{\text{power of } x}{\text{same power of } x} = \frac{\text{coefficient of largest } x}{\text{coefficient of largest } x}$
- $\lim_{x \rightarrow \pm\infty} \frac{\text{smaller power of } x}{\text{larger power of } x} = 0$
- $\lim_{x \rightarrow \pm\infty} \frac{\text{larger power of } x}{\text{smaller power of } x} = \pm\infty$
- Check every term for a higher power of x —most exams have at least one trick question to make sure that you are paying attention!
- Double-check whether your limit is to ∞ or to $-\infty$.

You will see this everywhere: $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ It is important to remember this. Similarly, $\frac{8}{x} = 8 \cdot 0 = 0$ and $\frac{1}{x} \cdot \frac{1}{x} = 0 \cdot 0 = 0$.

Common Cases:

- Both top and bottom have the same highest power of x :

$$\lim_{x \rightarrow \infty} \frac{x^3 + x}{3x^3} \rightarrow \frac{x^3 + x}{3x^3} \cdot \frac{1}{x^3} = \frac{\frac{x^3}{x^3} + \frac{x}{x^3}}{\frac{3x^3}{x^3}} = \frac{1 + \frac{1}{x^2}}{3} = \frac{1 + 0}{3} = \frac{1}{3} = \lim_{x \rightarrow \infty} \frac{1}{3} = \frac{1}{3}$$

*Be careful! $x + 5 + x^2 + x^4 = x^4 + x^2 + x + 5$ The highest power of x is not always the first.

- The top has a higher power:

$$\lim_{x \rightarrow -\infty} \frac{x^4 + 6}{x^3 - 4} \rightarrow \frac{x^4 + 6}{x^3 - 4} \cdot \frac{1}{x^3} = \frac{\frac{x^4}{x^3} + \frac{6}{x^3}}{\frac{x^3}{x^3} - \frac{4}{x^3}} = \frac{x + \frac{6}{x^3}}{1 - \frac{4}{x^3}} = \frac{x + 0 + 0}{1 - 0} \rightarrow \lim_{x \rightarrow -\infty} x = -\infty$$

- The bottom has a higher power of x :

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^3 - x + 2}{7x^5 - 3} &\rightarrow \frac{x^3 - x + 2}{7x^5 - 3} \cdot \frac{1}{x^5} = \frac{\frac{x^3}{x^5} - \frac{x}{x^5} + \frac{2}{x^5}}{\frac{7x^5}{x^5} - \frac{3}{x^5}} = \frac{\frac{1}{x^2} - \frac{1}{x^4} + \frac{2}{x^5}}{7 - \frac{3}{x^5}} = \frac{\frac{1}{x^2} - 0 + 0}{7 - 0} = \frac{1}{7x^2} \\ &\rightarrow \lim_{x \rightarrow \infty} \frac{1}{7x^2} = 0 \end{aligned}$$

(Turn Over)



Limits to Infinity

- With square roots:

$$\lim_{x \rightarrow \infty} \frac{2x^4 - 7}{\sqrt{4x^8 + 7x^5}} \rightarrow \frac{2x^4 - 7}{\sqrt{4x^8 + 7x^5}} \cdot \frac{1}{\frac{1}{\sqrt{x^8}}} = \frac{\frac{2x^4}{x^4} - \frac{7}{x^4}}{\sqrt{\frac{4x^8}{x^8} + \frac{7x^5}{x^8}}} = \frac{2 - 0}{\sqrt{4 + 0}} = \frac{2}{\sqrt{4}} = \frac{2}{2} = 1$$

