Subject: Business Math/Finance
Created by: Leo Perez
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Compound Interest and Time Value of Money

Compound Interest:

$$A = P(1+i/n)^{nt}$$

Compound interest (or compounding interest) is interest calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan. Compound Interest is also crucial to understanding Time Value of Money.

Variables:

A (Amount) – the amount in the account after t years.

P (Principal) – the original amount of an investment.

i (Interest Rate) - the charge for the privilege of borrowing money, typically expressed as annual percentage rate. This is can also be represented with the letter r.

n (Period(s)) - the number of intervals or periods that are being compounded yearly.

t (Time) – the amount of time to maturity of an investment in years.

Example (Compound Interest):

Suppose you deposit $1,000 in a bank that pays 8% quarterly. How much will the bank owe you at the end of a year?

Step 1: We must first identify all of our variables. In this case: P=$1,000, i=8%, n=4, t=1. We can now plug these into our equation. (Note: n is 4 because the bank is compounding quarterly.)

$$A=\$1,000(1+.08/4)^{4*1}$$

(Note: We want to represent i as a decimal so we divide 8% by 100%.)

Step 2: We can simplify .08/4 (i/n) and 4*1 (nt).

$$A=\$1,000(1+.02)^{4}$$

Step 3: Further simplify within the parenthesis.

$$A=\$1,000(1.02)^{4}$$

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Step 4: Raise 1.02 to the 4^{th} power.

$$A=\$1,000(1.08243216)$$

Step 5: Multiply $1,000 and 1.082…

$$A=\$1,082.43$$

Continuous Compounding:

If a principal P is invested at an annual rate r (expressed as a decimal) compounded continuously, then the amount A in the account at the end of t years is given by

$$A = Pe^{rt}$$

Example (Continuous Compounding):

If $23,000 is invested at 13.5% compounded continuously, what is the amount after 15 years?

Step 1: We must first identify all of our variables. In this case: P=$23,000, r=13.5%, t=15. We can now plug these into our equation.

$$A=\$23,000e^{.135*15}$$

Step 2: Simplify the exponent by multiplying .135 (13.5%) times 15 (t).

$$A=\$23,000e^{2.025}$$

Step 3: Simplify $e^{2.025}$.

$$A=\$23,000(7.576110945)$$

Step 4: Multiply $23,000 time 7.576…

$$A=\$174,250.55$$

Time Value of Money:

Definition: The time value of money (TVM) is the idea that money available at the present time is worth more than the same amount in the future due to its potential earning capacity.

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Formulas:

$$\text{Future Value}=\text{Present Value}(1+i/n)^{nt}$$

$$\text{Present Value}=\text{Future Value}/(1+i/n)^{nt}$$

$$\text{(Annuity) Future Value}=\text{Present Value}[[(1+i/n)^{nt}-1]/(i/n)]$$

$$\text{(Annuity) Present Value}=\text{Future Value}[(i/n)/[(1+i/n)^{nt}-1]$$

Annuity: An annuity is a contractual financial product sold by financial institutions that is designed to accept and grow funds from an individual and then, upon annuitization, pay out a stream of payments to the individual at a later point in time.

Example (Time Value of Money):

If you wish to accumulate $197,000 in 5 years, how much must you deposit today in an account that pays a quoted annual interest rate of 13% with semi-annual compounding of interest?

Step 1: We must first identify all of our variables. In this case: FV=$197,000, i=13%, n=2, t=5. We can now plug them into the equation.

$$PV=\$197,000/(1+.13/2)^{2*5}$$

Step 2: We can simplify .13/2 and 2*5.

$$PV= \$197,000/(1+.065)^{10}$$

Step 3: Simplify within the parenthesis.

$$PV= \$197,000/(1.065)^{10}$$

Step 4: Simplify $(1.065)^{10}$

$$A= \$197,000/1.877137465$$

Step 5: Divide
A= $104,947.03

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References: The following resources were referred to during the creation of this handout: Investopedia, University of West Georgia, and College Mathematics for Business, Economics, Life Sciences, and Social Sciences 13th Edition (Pearson).