

Mean, median, mode, variance & standard deviation

MEAN – average value of the data set

symbol(s): μ – population mean

\bar{x} – sample mean,
$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$$

Example 1: Calculate the mean for the data set: 1, 4, 4, 6, 10.

$$\bar{x} = \frac{\sum_{i=1}^5 x_i}{5} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{1 + 4 + 4 + 6 + 10}{5} = \frac{25}{5} = 5$$

MEDIAN – “the middle number”, a number that splits data set in half

How to find median?

Step 1: Arrange data in increasing order

Step 2: Determine how many numbers are in the data set = n

Step 3: If n is odd: Median is the middle number

If n is even: Median is the average of two middle numbers

Example 2: Find the median for the data set: 34, 22, 15, 25, 10.

Step 1: Arrange data in increasing order 10, 15, 22, 25, 34

Step 2: There are 5 numbers in the data set, n = 5.

Step 3: n = 5, so n is an odd number Median = middle number, median is 22.

Example 3: Find the median for the data set: 19, 34, 22, 15, 25, 10.

Step 1: Arrange data in increasing order 10, 15, 19, 22, 25, 34

Step 2: There are 6 numbers in the data set, n = 6.

Step 3: n = 6, so n is an even number Median = average of two middle numbers

$$\text{median} = \frac{19 + 22}{2} = 20.5$$

Notes: Mean and median don't have to be numbers from the data set!

Mean and median can only take one value each.

Mean is influenced by extreme values, while median is resistant.



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MODE – The most frequent number in the data set

Example 4: Find the mode for the data set: 19, 19, 34, 3, 10, 22, 10, 15, 25, 10, 6.

The number that occurs the most is number 10, mode = 10.

Example 5: Find the mode for the data set: 19, 19, 34, 3, 10, 22, 10, 15, 25, 10, 6, 19.

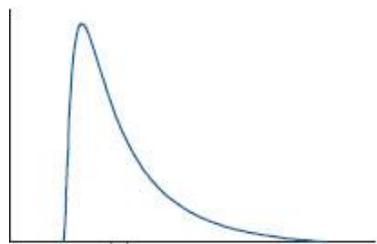
Number 10 occurs 3 times, but also number 19 occurs 3 times, since there is no number that occur 4 times both numbers 10 and 19 are mode, mode = {10, 19}.

Notes: Mode is always the number from the data set.

Mode can take zero, one, or more than one values. (There can be zero modes, one mode, two modes, ...)

Shape of a Data Set

Relationship between mean and median, in most cases, can describe the shape of data set (histogram).



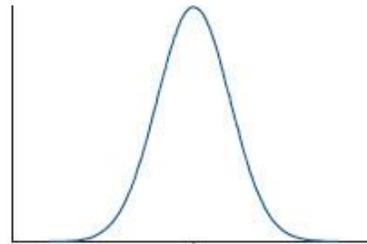
Median Mean

(a)

(a) Median < Mean

skewed to the RIGHT

positively skewed

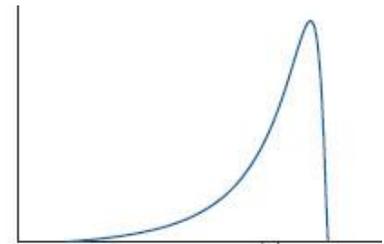


Mean = Median

(b)

(b) Median = Mean

symmetric



Mean Median

(c)

(c) Median > Mean

skewed to the LEFT

negatively skewed



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Mean, median, mode, variance & standard deviation

The difference between value x and population mean μ , $x - \mu$ is **deviation**.

VARIANCE measures how far the values of the data set are from the mean, on average. The average of the squared deviations is the population variance.

Symbol(s): σ^2 – population variance

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

s^2 – sample variance

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

STANDARD DEVIATION – square root of the variance

Symbol(s): σ – population standard deviation

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

s – sample standard deviation

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

How to compute variance and standard deviation? (of a sample)

For variance do steps 1-5, for standard deviation do steps 1-6.

Step 1 – Compute the sample mean \bar{x}

Step 2 – Calculate the difference of $x_i - \bar{x}$, for each value in the data set

Step 3 – Calculate the squared difference $(x_i - \bar{x})^2$, for each value in the data set

Step 4 – Sum the squared differences $\sum_{i=1}^n (x_i - \bar{x})^2$

Step 5 – Divide the sum of squared differences with $n-1$, $\text{variance} = s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$

Step 6 – ONLY FOR STANDARD DEVIATION

Calculate the squared root of variance, $\text{standard deviation} = \sqrt{\text{variance}}$

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

References: *Essential Statistics, 2nd Ed., Navidi, Monk*



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