

## Integration By Parts

Integration by parts is a technique for performing indefinite integration  $\int u dv$  or definite integration  $\int_a^b u dv$  by expanding the differential of a product of functions  $d(uv)$  and expressing the original integral in terms of a known integral  $\int v du$ . A single integration by parts starts with

$$d(uv) = u dv + v du, \quad (1)$$

and integrates both sides,

$$\int d(uv) = uv = \int u dv + \int v du. \quad (2)$$

Rearranging gives

$$\int u dv = uv - \int v du. \quad (3)$$

For example, consider the integral  $\int x \cos x dx$  and let

$$u = x \quad dv = \cos x dx \quad (4)$$

$$du = dx \quad v = \sin x, \quad (5)$$

so integration by parts gives

$$\int x \cos x dx = x \sin x - \int \sin x dx \quad (6)$$

$$= x \sin x + \cos x + C, \quad (7)$$

where  $C$  is a [constant of integration](#).

The procedure does not always succeed, since some choices of  $u$  may lead to more complicated integrals than the original. For example, consider again the integral  $\int x \cos x dx$  and let

$$u = \cos x \quad dv = x dx \quad (8)$$

$$du = -\sin x dx \quad v = \frac{1}{2} x^2,$$



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giving

$$\int x \cos x \, dx = \frac{1}{2} x^2 \cos x - \frac{1}{2} \int x^2 (-\sin x) \, dx \quad (9)$$

$$= \frac{1}{2} x^2 \cos x + \frac{1}{2} \int x^2 \sin x \, dx, \quad (10)$$

which is more difficult than the original (Apostol 1967, pp. 218-219).

Integration by parts may also fail because it leads back to the original integral. For example, consider  $\int x^{-1} \, dx$  and let

$$\begin{aligned} u &= x & dv &= x^{-2} \, dx \\ du &= dx & v &= -x^{-1}, \end{aligned} \quad (11)$$

then

$$\int x^{-1} \, dx = -1 - \int (-x^{-1}) \, dx + C = \int x^{-1} \, dx + C - 1, \quad (12)$$

which is same integral as the original (Apostol 1967, p. 219).

The analogous procedure works for definite integration by parts, so

$$\int_a^b u \, dv = [uv]_a^b - \int_a^b v \, du, \quad (13)$$

where  $[f]_a^b = f(b) - f(a)$ .

Problems-

*PROBLEM 1* : Integrate  $\int x e^x \, dx$

*PROBLEM 2* : Integrate  $\int x \sin x \, dx$

*PROBLEM 3* : Integrate  $\int x \ln x \, dx$

*PROBLEM 4* : Integrate  $\int x \cos 3x \, dx$

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**References:** The following works were referred to during the creation of this handout: [D.A. Kouba's \*Integration by Parts\*](#).

