Subject: Calculas
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Revised: 07/10/2018

Rates of Change and Derivatives

1. **Average Rate of Change:** The average rate of change is given by the change in the “y” values over the change in the “x” values.

   For $y = f(x)$, the average rate of change from $x = a$ to $x = a + h$ is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \quad h \neq 0$$

2. **Instantaneous Rate of Change:** The instantaneous rate of change is given by the slope of a function $f(x)$ evaluated at a single point $x = a$.

   For $y = f(x)$, the instantaneous rate of change at $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{if the limit exists}$$

3. **Derivative:** The derivative of a function represents an infinitesimal change in the function with respect to one of its variables. It is also represented by the slope of the tangent like at a particular point for the function curve. The "simple" derivative of a function f with respect to a variable x is $\frac{dy}{dx}$, also denoted as $f'(x)$.

   Here are some ways to find the derivative of a function:

   a. **Using the Definition of the Derivative**

   For $y = f(x)$, we define the derivative of f at x, denoted $f'(x)$,by

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If $f'(x)$ exists for each x in the open interval (a,b),then f is said to be differentiable over (a,b).

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For example:

Find the derivative of $f(x) = x^2 + x$ using the definition of derivative

Solution: $f(x+h) = (x+h)^2 + (x+h)$

$f(x) = x^2 + x$

$$f'(x) \equiv \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}, = \lim_{h\to 0} \frac{2x+2h+1-2x-1}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 \; + (x+h) - (x^2+x)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2+h}{h}$$

$$= \lim_{h\to 0} 2x + h + 1$$

Set $h = 0$

$f'(x) = 2x + 1$

b. "Short-cut" way— using already defined formulas

For Example: Given the power rule $\frac{dy}{dx} x^n = nx^{n-1}$

Find the derivative of $f(x) = x^2 + x$.

Solution: $f'(x) = 2x^{2-1} + x^{1-1}$

$= 2x + 1$ (Note that $x^0 = 1$)

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NOTE: For more formulas, refer to the Differentiation and Integration Formulas handout.

Here are some examples where the derivative ass the slope of the tangent can be applied:

Find an equation of the tangent line to the curve at the given point.

$$y = 2x^3 - x^2 + 2 \quad ; \quad (1,3)$$

- First, find the derivative: $y' = 6x^2 - 2x$

  ->remember that derivative equals slope, 'm.'

- Secondly, plug in the x-value from the point given, (1,3) into the derivative.

$$y' = 6(1)^2 - 2(1) = 4$$

therefore, **m=4**

- Thirdly, use point-slope formula to find the tangent line.

Point-slope formula: $y - y_1 = m\,(x - x_1)$ $\qquad (1, 3) = (x_1, y_1)$

$$y - 3 = 4(x - 1)$$

$$y - 3 = 4x - 4$$

$$\boxed{y = 4x - 1}$$ **--this is the tangent line.**

1) Find the equation of the normal line to the curve at the given point. (1,3)

-> Normal line means perpendicular line.

- The slope, 'm' of a perpendicular line is the negative reciprocal, for example, if an equation has slope, $m = \frac{2}{3}$, it's perpendicular line will have slope, $m = -\frac{3}{2}$.

- Since the our given equation had slope, $m = \frac{3}{1}$, it's normal line will have slope, $m = -\frac{1}{3}$.

Now, use point slope formula again using slope, $\mathbf{m = -\frac{1}{3}}$.

Point-slope formula : $y - y_1 = m\,(x - x_1)$ $\qquad (1, 3) = (x_1, y_1)$

$$y - 3 = -\frac{1}{3}(x - 1)$$

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$$\boxed{y - 3 = -\frac{1}{3}x + \frac{1}{3}}$$

this is the normal line

y-3=-1/3(x-1)

y-3=-1/3x+1/3

The following works were referred to during the creation of this handout: *Stewart Calculus, 8th Ed.*