

# Word Problems: Mixture, Uniform, and Work Problems

## I. Mixture Problems

Identify if it is a mixture problem: Mixture problems involves **combining/mixing** two ingredients/substances that have different characteristics and **prices or percent** into one **mixture**.

**First step:** Remember these formulas:

$$\text{Amount} * \text{Cost (\$)} = \text{Value (A*C=V)} \text{ or}$$

$$\text{Amount} * \text{Percent (\%)} = \text{Value (A * P = V)}$$

Amount of the mixture is the sum of the amount of 2 ingredients/substances

Value of the mixture is the sum of the values of 2 ingredients/substances

**HOWEVER**, cost or percent of the mixture is **NOT** the sum of the cost of 2 ingredients

**Second Step:** Set up a table: let the unknown variable be  $x$  → fill out the table based on the given information and  $x$  → Set up an equation based on the table → Solve for  $x$

**Example:** How many pounds of peanuts that cost \$2.25 per pound must be mixed with 40lb of cashews that cost \$6.00 per pound to make a mixture that costs \$3.50 per pound?

We are given: Cost of peanuts = \$2.25; Cost of cashews = \$6.00; Cost of mixture= \$3.50;  
Amount of cashews = 40 lbs.

Let the amount of peanuts (what we are looking for) be  $x$

→ Amount of mixture = amount of peanuts + amount of cashews =  $x + 40$

	Amount (A)	Cost (C)	Value (V)
Peanuts	$x$	2.25	$2.25x$
Cashews	<b>40</b>	6.00	<b>6.00 (40)</b>
Mixture	$x + 40$	3.50	<b>3.50 (x+4)</b>

Because value of mixture = value of cashews + value of peanuts, we can get the following equation:

$$2.25x + 6.00(40) = 3.50(x+40)$$

$$2.25x + 240 = 3.50x + 140 \text{ (Subtract } 3.50x \text{ on both sides)}$$

$$-1.25x = -100 \rightarrow x = \text{the amount of peanuts} = \mathbf{80 \text{ pounds}}$$



# Word Problems: Mixture, Uniform, and Work Problems

## II. Uniform Motion/ Rate Problems

Identify if it is uniform motion/ rate problem. An object that moves at a constant rate is said to be uniform motion. Key words: **rate, time, and distance**

**First Step:** Remember the formulas:

$$\text{Rate} * \text{Time} = \text{Distance} (r*t = d)$$

$$\rightarrow t = d/r \text{ and } r = d/t$$

**Second step:** Set up a table: let the unknown variable be  $x$   $\rightarrow$  fill out the table based on the given information and  $x$   $\rightarrow$  Set up an equation based on the table  $\rightarrow$  Solve for  $x$

**Example:** *Two planes are 1620 miles apart and are traveling toward each other. One plane is travelling 120 mph faster than the other plane. The planes meet in 1.5 h. Find the speed of each plane.*

We are given: total distance = 1620 miles = distance plane 1 travels + distance plane 2 travels

$$\text{Time plane 1 travels} = \text{Time plane 2 travels} = 1.5 \text{ h}$$

We are looking for: Speed or rate for each plane given that one is 120 mph faster than the other

Let  $x$  be the rate of plane 1 (slower plane)

$\rightarrow$  The rate of plane 2 (faster plane) =  $x + 120$

	Rate (r)	Time (t)	Distance
Plane 1	$x$	1.5	$1.5x$
Plane 2	$x + 120$	1.5	$1.5 (x+120)$

Total distance = 1620 miles = distance plane 1 travels + distance plane 2 travels, we have equation:

$$1.5x + 1.5 (x+120) = 1620$$

$$1.5x + 1.5x + (1.5) 120 = 1620 \text{ (Distribute 1.5 into } x \text{ and } 120)$$

$$3x + 180 = 1620 \text{ (Combine like terms)}$$

$$3x = 1440 \text{ (Subtract 180 on both sides)}$$

$$x = 480 \rightarrow \text{Plane 1 (slower plane) travels at the speed 480 mph}$$

$$\text{The rate of plane 2 (faster plane)} = x + 120 = 480 + 120 = 600 \text{ mph}$$



# Word Problems: Mixture, Uniform, and Work Problems

## III. Work problems

Identify if it is work problem. Key words: **alone, together.**

In a work problem, the rates at which certain persons or machines work alone are usually given, and it is necessary to compute the rate at which they work together (or vice versa)

**First Step:** Remember the formula:

$$\frac{1}{r} + \frac{1}{s} = \frac{1}{t}$$

In which: r, s are, for example, the number of hours it takes Rae and Sam, respectively to complete a job when working alone

$\frac{1}{r}, \frac{1}{s}$  Jobs done by Rae and Sam in 1 hour respectively

t is the total number of hours that it takes Rae and Sam to complete a job together.

**Example:** If Machine X can produce 1000 bolts in 4 hours and Machine Y can produce 1000 bolts in 5 hours, in how many hours can Machines X and Y, working together at these constant rates, produce 1000 bolts?

Given: Machine X can finish the job alone in 4 hours; Machine Y can finish the job alone in 5 hours

Let t = number of hours two machines work together to complete the job, we have:

$$\frac{1}{4} + \frac{1}{5} = \frac{1}{t}$$

$$\frac{5}{20} + \frac{4}{20} = \frac{1}{t}$$

$$\frac{9}{20} = \frac{1}{t}$$

$$9t = 20$$

$$t = 20/9$$

--



# Word Problems: Mixture, Uniform, and Work Problems

## More Examples:

**Example 1.**  $ax \pm b = c$ . All problems similar to the following lead eventually to an equation in that simple form:

Jane spent \$42 for shoes. This was \$14 less than twice what she spent for a blouse. How much was the blouse?

**Solution.** Let  $x$ , then, be how much she spent for the blouse. The problem states that "This" -- that is, \$42 -- was \$14 less than two times  $x$ .

Here is the equation:  $2x-14=12$

The blouse cost \$28.

**Example 2.** There are  $b$  boys in the class. This is three more than four times the number of girls. How many girls are in the class?

**Solution.** Again, let  $x$  represent the unknown number that you are asked to find: Let  $x$  be the number of girls.

(Although  $b$  is not known, it is not what you are asked to find.)

The problem states that "This" --  $b$  -- is three more than four times  $x$ :

$$4x+3=b$$

The solution here is not a number, because it will depend on the value of  $b$ . This is a type of "literal" equation, which is very common in algebra.

**Example 3. The whole is equal to the sum of the parts.** The sum of two numbers is 84, and one of them is 12 more than the other. What are the two numbers?

**Solution.** In this problem, we are asked to find two numbers. Therefore, we must let  $x$  be one of them. Let  $x$ , then, be the first number.

We are told that the other number is 12 more,  $x + 12$ .

The problem states that their sum is 84:

$$x + \overline{x + 12} = 84$$

The line over  $x + 12$  is a grouping symbol called a *vinculum*. It saves us writing parentheses.

We have:  $2x=84-12$

This is the first number. Therefore the other number is

$$x + 12 = 36 + 12 = 48.$$

The sum of  $36 + 48$  is 84.



(510) 885-3674

[www.csueastbay.edu/scaa](http://www.csueastbay.edu/scaa)

[scaa@csueastbay.edu](mailto:scaa@csueastbay.edu)

## Word Problems: Mixture, Uniform, and Work Problems

**Example 4.** The sum of two consecutive numbers is 37. What are they?

**Solution.** Two consecutive numbers are like 8 and 9, or 51 and 52.

Let  $x$ , then, be the first number. Then the number after it is  $x + 1$ .

The problem states that their sum is 37:

$$x + \overline{x + 1} = 37$$

The two numbers are 18 and 19.

**Example 5.** One number is 10 more than another. The sum of twice the smaller plus three times the larger, is 55. What are the two numbers?

**Solution.** Let  $x$  be the smaller number.

Then the larger number is 10 more:  $x + 10$ .

The problem states:

$$2x + 3(x + 10) = 15$$

That's the smaller number  $x=5$ . The larger number is 10 more: 15.

**Example 6.** Divide \$80 among three people so that the second will have twice as much as the first, and the third will have \$5 less than the second.

**Solution.** Again, we are asked to find more than one number. We must begin by letting  $x$  be how much the first person gets.

Then the second gets twice as much,  $2x$ .

And the third gets \$5 less than that,  $2x - 5$ .

Their sum is \$80:

$$x + 2x + \overline{2x - 5} = 80$$

This is how much the first person gets. Therefore, the second gets

$$2x = 34.$$

And the third gets

$$2x - 5 = 29.$$

The sum of 17, 34, and 29 is in fact 80.

### References -

The following works were referred to during the creation of this handout: Basic Mathematics' *Algebra Word Problems*, Wolfram Alpha's *Mathematical Word Problems*, and Super Teacher Worksheets' *Math Word Problem Workshop*, *The Official Guide for GMAT Review Quantitative Review* (Wiley, 2015)



(510) 885-3674

[www.csueastbay.edu/scaa](http://www.csueastbay.edu/scaa)

[scaa@csueastbay.edu](mailto:scaa@csueastbay.edu)