Calculus Reference Sheet

Derivatives

(1) $\frac{d}{dx}[c] = [0]\,u'$

(2) $\frac{d}{dx}[cf(u)] = [cf'(u)]\,u'$

(3) $\frac{d}{dx}[f(u)+g(u)] = [f'(u)+g'(u)]\,u'$

(4) $\frac{d}{dx}[f(u)\cdot g(u)] = [f'(u)\cdot g(u) + f(u)\cdot g'(u)]\,u'$

(5) $\frac{d}{dx}\left[\frac{f(u)}{g(u)}\right] = \left[\frac{f'(u)\cdot g(u) - f(u)\cdot g'(u)}{[g(u)]^2}\right]u'$

(6) $\frac{d}{dx}[u^n] = \left[nu^{n-1}\right]u'$

(7) $\frac{d}{dx}[\ln u] = \left[\frac{1}{u}\right]u'$

(8) $\frac{d}{dx}[\log_a u] = \left[\frac{1}{u\ln a}\right]u'$

(9) $\frac{d}{dx}[e^u] = [e^u]\,u'$

(10) $\frac{d}{dx}[a^u] = [a^u \ln a]\,u'$

(11) $\frac{d}{dx}[\sin u] = [\cos u]\,u'$

(12) $\frac{d}{dx}[\cos u] = [-\sin u]\,u'$

(13) $\frac{d}{dx}[\tan u] = \left[\sec^2 u\right]u'$

(14) $\frac{d}{dx}[\cot u] = \left[-\csc^2 u\right]u'$

(15) $\frac{d}{dx}[\sec u] = [\sec u\tan u]\,u'$

(16) $\frac{d}{dx}[\csc u] = [-\csc u\cot u]\,u'$

(17) $\frac{d}{dx}[\arcsin u] = \left[\frac{1}{\sqrt{1-u^2}}\right]u'$

(18) $\frac{d}{dx}[\arccos u] = \left[\frac{-1}{\sqrt{1-u^2}}\right]u'$

(19) $\frac{d}{dx}[\arctan u] = \left[\frac{1}{1+u^2}\right]u'$

(20) $\frac{d}{dx}[\text{arccot}\ u] = \left[\frac{-1}{1+u^2}\right]u'$

(21) $\frac{d}{dx}[\text{arcsec}\ u] = \left[\frac{1}{|u|\sqrt{u^2-1}}\right]u'$

(22) $\frac{d}{dx}[\text{arccsc}\ u] = \left[\frac{-1}{|u|\sqrt{u^2-1}}\right]u'$

Integrals

(1) $\int kf(u)\,du = k\int f(u)\,du$

(2) $\int f(u)\pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du$

(3) $\int u^a\,du = \frac{u^{a+1}}{a+1} + C, \quad a \neq -1$

(4) $\int \frac{1}{u}\,du = \ln|u| + C$

(5) $\int e^u\,du = e^u + C$

(6) $\int a^u\,du = \frac{1}{\ln a}a^u + C$

(7) $\int \sin u\,du = -\cos u + C$

(8) $\int \cos u\,du = \sin u + C$

(9) $\int \sec^2 u\,du = \tan u + C$

(10) $\int \csc^2 u\,du = -\cot u + C$

(11) $\int \tan u\,du = -\ln|\cos u| + C$

(12) $\int \cot u\,du = \ln|\sin u| + C$

(13) $\int \sec u\,du = \ln|\sec u + \tan u| + C$

(14) $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

(15) $\int \sec u\ \tan u\,du = \sec u + C$

(16) $\int \csc u\ \cot u\,du = -\csc u + C$

(17) $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\frac{u}{a} + C$

(18) $\int \frac{1}{\sqrt{u^2\pm a^2}}\,du = \ln\left[u + \sqrt{u^2\pm a^2}\right] + C$

(19) $\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\text{arcsec}\,\frac{|u|}{a} + C$

(20) $\int \frac{1}{u\sqrt{a^2\pm u^2}}\,du = \frac{-1}{a}\ln\frac{a+\sqrt{a^2\pm u^2}}{|u|} + C$

(21) $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a} + C$

(22) $\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

Projectile Motion

$$x = x_0 + (v_0 \cos\alpha)\, t \qquad y = y_0 + (v_0 \sin\alpha)\, t - \frac{1}{2} g t^2$$

Vector Functions

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||} \qquad a_T = \mathbf{a}\cdot\mathbf{T} = \frac{d^2 s}{dt^2} = \frac{\mathbf{a}\cdot\mathbf{v}}{||\mathbf{v}||} \qquad a_N = \mathbf{a}\cdot\mathbf{N} = \kappa \left(\frac{ds}{dt}\right)^2 = \frac{||\mathbf{a}\times\mathbf{v}||}{||\mathbf{v}||}$$

Curvature

$$\kappa = \frac{|y''|}{\left[1 + (y')^2\right]^{3/2}} \quad \text{or} \quad \frac{|x'y'' - x''y'|}{\left[(x')^2 + (y')^2\right]^{3/2}} \quad \text{or} \quad \frac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||} \quad \text{or} \quad \frac{||\mathbf{r}'(t)\times\mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3} \quad \text{or} \quad \frac{|\mathbf{a}(t)\cdot\mathbf{N}(t)|}{||\mathbf{v}(t)||^2}$$

Polar Coordinates

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x} \qquad dA = r\, dr\, d\theta$$

Spherical Coordinates

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$x = \rho\sin\phi\cos\theta$$

$$y = \rho\sin\phi\sin\theta \qquad \cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$z = \rho\cos\phi$$

$$\tan\theta = \frac{y}{x}$$

Vector Fields

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} \qquad \text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$$

Green's Theorem

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

Divergence Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{N}\, dS = \iiint_Q \text{div}(\mathbf{F})\, dV$$

Stokes' Theorem

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \text{curl } (\mathbf{F})\cdot\mathbf{N}\, dS$$