Convergence Tests

Name	Series	Converges	Diverges	Other
kth term (Divergence)	$\sum_{k=1}^{\infty} a_k$		$\lim_{k\to\infty} a_k \neq 0$	cannot be used for convergence
geometric	$\sum_{k=0}^{\infty} ar^k$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	$S = \frac{a}{1-r}$
telescoping	$\sum_{k=1}^{\infty} (a_k - a_{k-1})$	$\lim_{k\to\infty} S_k = L$		S can be found by expanding
p-series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	$p > 1$	$p \leq 1$	$0 < R_N < \frac{N^{1-p}}{p-1}$
alternating	$\sum_{k=1}^{\infty} (-1)^k a_k$	a_k decrease, approach 0		$\lvert R_N\rvert < a_{N+1}$
integral	$\sum_{k=1}^{\infty} a_k, \quad a_k = f(k)$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	$0 < R_N < \int_N^{\infty} f(x)\,dx$
ratio	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \left\lvert \frac{a_{k+1}}{a_k} \right\rvert < 1$	$\lim_{k\to\infty} \left\lvert \frac{a_{k+1}}{a_k} \right\rvert > 1$	
root	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert} < 1$	$\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert} > 1$	
direct comparison	$\sum_{k=1}^{\infty} a_k$	$0 < a_k < b_k$ and $\sum_{k=1}^{\infty} b_k$ converges	$0 < b_k < a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges	
limit comparison	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \frac{a_k}{b_k} = L > 0$ and $\sum_{k=1}^{\infty} b_k$ converges	$\lim_{k\to\infty} \frac{a_k}{b_k} = L > 0$ and $\sum_{k=1}^{\infty} b_k$ diverges	L must be a finite number.

Series

Function	Series	Closed Form	Interval of convergence
$\frac{1}{u}$	$1-(u-1)+(u-1)^2-(u-1)^3+\dots$	$\sum_{k=0}^{\infty}(-1)^k(u-1)^k$	$0<u<2$
$\frac{1}{1-u}$	$1+u+u^2+u^3+\dots$	$\sum_{k=0}^{\infty}u^k$	$-1<u<1$
$\ln u$	$(u-1)-\frac{(u-1)^2}{2}+\frac{(u-1)^3}{3}\dots$	$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}(u-1)^k}{k}$	$0<u\leq 2$
e^u	$1+u+\frac{u^2}{2!}+\frac{u^3}{3!}+\dots$	$\sum_{k=0}^{\infty}\frac{u^k}{k!}$	$-\infty<u<\infty$
$\cos u$	$1-\frac{u^2}{2!}+\frac{u^4}{4!}-\frac{u^6}{6!}+\dots$	$\sum_{k=0}^{\infty}\frac{(-1)^k u^{2k}}{(2k)!}$	$-\infty<u<\infty$
$\sin u$	$u-\frac{u^3}{3!}+\frac{u^5}{5!}-\frac{u^7}{7!}+\dots$	$\sum_{k=0}^{\infty}\frac{(-1)^k u^{2k+1}}{(2k+1)!}$	$-\infty<u<\infty$
$\arctan u$	$u-\frac{u^3}{3}+\frac{u^5}{5}-\frac{u^7}{7}\dots$	$\sum_{k=0}^{\infty}\frac{(-1)^k u^{2k+1}}{2k+1}$	$-1\leq u\leq 1$
$\arcsin u$	$u+\frac{u^3}{2\cdot 3}+\frac{1\cdot 3u^5}{2\cdot 4\cdot 5}+\frac{1\cdot 3\cdot 5u^7}{2\cdot 4\cdot 6\cdot 7}+\dots$	$\sum_{k=0}^{\infty}\frac{(2k)!\,u^{2k+1}}{(2^k k!)^2(2k+1)}$	$-1\leq u\leq 1$
$(1+u)^k$	$1+ku+\frac{k(k-1)u^2}{2!}+\frac{k(k-1)(k-2)u^3}{3!}+\cdots$		$-1<u<1$
$(1+u)^{-k}$	$1-ku+\frac{k(k+1)u^2}{2!}-\frac{k(k+1)(k+2)u^3}{3!}+\cdots$		$-1<u<1$