Pre-Calculus Reference Sheet

Factoring

- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + b^2$ is prime
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$
- $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

Analytic Geometry

- slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- equation of a line: $y - y_1 = m\,(x - x_1)$
- distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Exponent Rules

- $a^{x+y} = a^x a^y$
- $(ab)^x = a^x b^x$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$ if $a \neq 0$
- $a^{-x} = \dfrac{1}{a^x}$ if $a \neq 0$
- $a^{x-y} = \dfrac{a^x}{a^y}$ if $a \neq 0$

Logarithm Rules

- $\log_b x = y \iff x = b^y$
- $b^{\log_b x} = x$
- $\log_b b^x = x$
- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b xy = \log_b x + \log_b y$
- $\log_b \dfrac{x}{y} = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- $\log_b x = \dfrac{\log_a x}{\log_a b}$

Arithmetic Series

- $a_k = a + (k-1)d$
- $S_n = \sum_{k=1}^{n} [a + (k-1)d] = \dfrac{n}{2}\,[2a + (n-1)d]$
- $S_n = \sum_{k=1}^{n} [a + (k-1)d] = n\left(\dfrac{a + a_n}{2}\right)$

Geometric Series

- $a_n = ar^{n-1}$
- $S_n = \sum_{k=0}^{n-1} ar^k = a\left[\dfrac{1 - r^n}{1 - r}\right]$ if $r \neq 1$
- $S = \sum_{k=0}^{\infty} ar^k = \dfrac{a}{1 - r}$ if $|r| < 1$

Trigonometry

- $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
- $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Pythagorean Identities

- $\cos^2 A + \sin^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$

Ratio Identities

- $\tan A = \dfrac{\sin A}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sin A}$

Reciprocal Identities

- $\sec A = \dfrac{1}{\cos A}$ • $\csc A = \dfrac{1}{\sin A}$ • $\cot A = \dfrac{1}{\tan A}$

Sum and Difference Identities

- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Identities

- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$
- $\sin 2A = 2\cos A \sin A$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Half Angle Identities

- $\cos \dfrac{A}{2} = \pm\sqrt{\dfrac{1 + \cos A}{2}}$
- $\sin \dfrac{A}{2} = \pm\sqrt{\dfrac{1 - \cos A}{2}}$
- $\tan \dfrac{A}{2} = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$

Triple Angle Identities

- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\sin 3A = 3\sin A - 4\sin^3 A$

Power Reduction Identities

- $\cos^2 A = \dfrac{1 + \cos 2A}{2}$
- $\sin^2 A = \dfrac{1 - \cos 2A}{2}$
- $\tan^2 A = \dfrac{1 - \cos 2A}{1 + \cos 2A}$
- $\cos^3 A = \dfrac{3\cos A + \cos 3A}{4}$
- $\sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$

Sum-to-Product Identities

- $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
- $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

Product-to-Sum Identities

- $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
- $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$
- $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

Sums of Sines and Cosines

- $A\cos x + B\sin x = \sqrt{A^2+B^2}\sin(x+\phi)$ where $\cos\phi = \dfrac{B}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{A}{\sqrt{A^2+B^2}}$
- $A\cos x + B\sin x = \sqrt{A^2+B^2}\cos(x-\phi)$ where $\cos\phi = \dfrac{A}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{B}{\sqrt{A^2+B^2}}$

Laws of Sines and Cosines

- $c^2 = a^2 + b^2 - 2ab\cos C$
- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a Triangle

For a triangle with sides a, b, c and angles $\angle A$, $\angle B$, and $\angle C$,

- Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$
- Area $= \dfrac{1}{2}ab\sin C$
- Area $= \dfrac{c^2 \sin A \sin B}{2\sin C}$

Circular Section

- Arc length: $s = r\theta$
- Area: $A = \dfrac{1}{2}r^2\theta$
