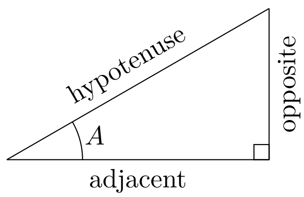
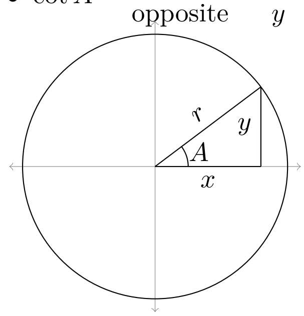
Trigonometry Reference Sheet

Definitions

- $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{x}{r}$
- $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{y}{r}$
- $\tan A = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{y}{x}$

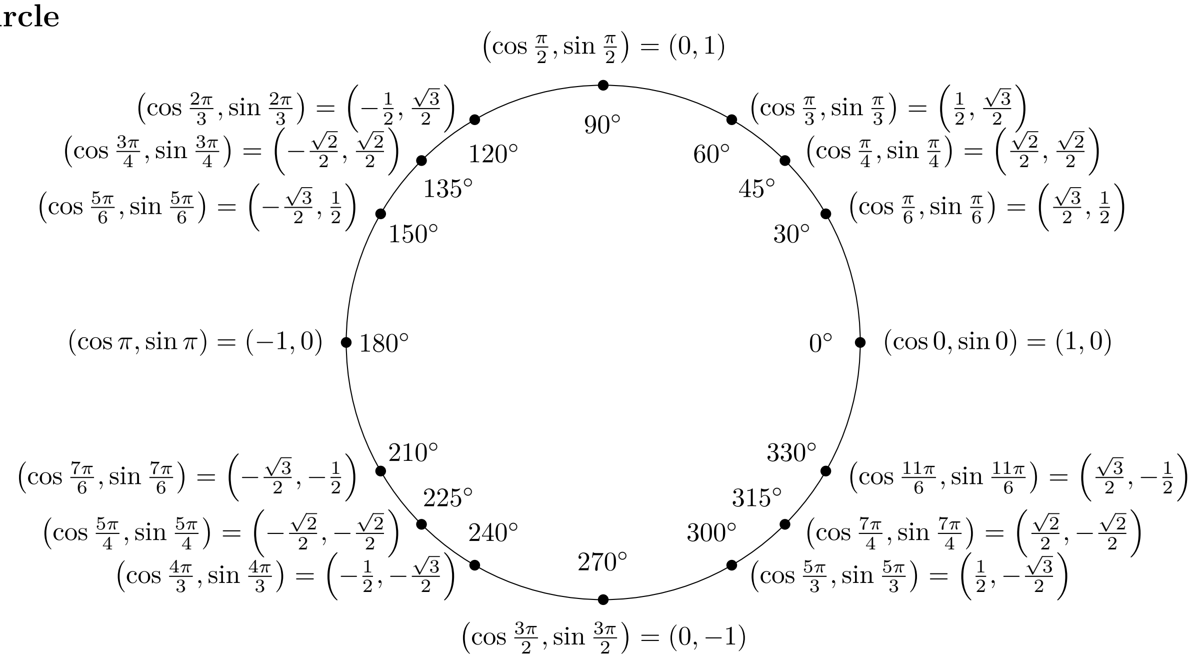
- $\sec A = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{r}{x}$
- $\csc A = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{r}{y}$
- $\cot A = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{x}{y}$





	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Unit Circle



Pythagorean Identities

- $\cos^2 A + \sin^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$

Reciprocal Identities

- $\sec A = \dfrac{1}{\cos A}$
- $\csc A = \dfrac{1}{\sin A}$
- $\cot A = \dfrac{1}{\tan A}$

Ratio Identities

- $\tan A = \dfrac{\sin A}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sin A}$

Cofunction Identities

- $\cos\left(\frac{\pi}{2} - A\right) = \sin A$
- $\sin\left(\frac{\pi}{2} - A\right) = \cos A$
- $\tan\left(\frac{\pi}{2} - A\right) = \cot A$
- $\sec\left(\frac{\pi}{2} - A\right) = \csc A$
- $\csc\left(\frac{\pi}{2} - A\right) = \sec A$
- $\cot\left(\frac{\pi}{2} - A\right) = \tan A$

Even/Odd Identities

- $\cos(-A) = \cos A$
- $\sin(-A) = -\sin A$
- $\tan(-A) = -\tan A$
- $\sec(-A) = \sec A$
- $\csc(-A) = -\csc A$
- $\cot(-A) = -\cot A$

Sum and Difference Identities

- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Identities

- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$
- $\sin 2A = 2\cos A \sin A$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Half Angle Identities

- $\cos \dfrac{A}{2} = \pm\sqrt{\dfrac{1 + \cos A}{2}}$
- $\sin \dfrac{A}{2} = \pm\sqrt{\dfrac{1 - \cos A}{2}}$
- $\tan \dfrac{A}{2} = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$

Power Reduction Identities

- $\cos^2 A = \dfrac{1 + \cos 2A}{2}$
- $\sin^2 A = \dfrac{1 - \cos 2A}{2}$
- $\tan^2 A = \dfrac{1 - \cos 2A}{1 + \cos 2A}$

Sum-to-Product Identities

- $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
- $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
- $\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

Product-to-Sum Identities

- $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
- $\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$
- $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

Sums of Sines and Cosines

- $A\cos x + B\sin x = \sqrt{A^2+B^2}\,\sin(x+\phi)$ where $\cos\phi = \dfrac{B}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{A}{\sqrt{A^2+B^2}}$
- $A\cos x + B\sin x = \sqrt{A^2+B^2}\,\cos(x-\phi)$ where $\cos\phi = \dfrac{A}{\sqrt{A^2+B^2}}$ and $\sin\phi = \dfrac{B}{\sqrt{A^2+B^2}}$

Circular Sections

- Arc length: $s = r\theta$
- Area: $A = \dfrac{1}{2}r^2\theta$
- Angular velocity: $\omega = \dfrac{v}{r}$

Graphing

For $y = A\cos(Bx + C) + D$ and $y = \sin(Bx + C) + D$,

- Amplitude $= |A|$
- Frequency $= B$
- Vertical Shift $= D$
- Period $= \dfrac{2\pi}{B}$
- Phase Shift $= \dfrac{-C}{B}$

The graph of $y = \tan(Bx + C)$ has asymptotes at the locations where $\cos(Bx + C) = 0$.

Range of Inverse Functions

- $y = \arccos x, \quad 0 \le y \le \pi$
- $y = \arcsin x, \quad -\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$
- $y = \arctan x, \quad -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$
- $y = \operatorname{arcsec} x, \quad 0 \le y < \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} < y \le \pi$
- $y = \operatorname{arccsc} x, \quad -\dfrac{\pi}{2} \le y < 0$ and $0 < y \le \dfrac{\pi}{2}$
- $y = \operatorname{arccot} x, \quad -\dfrac{\pi}{2} < y < 0$ and $0 < y < \dfrac{\pi}{2}$

Laws of Sines and Cosines

- $c^2 = a^2 + b^2 - 2ab\cos C$
- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
